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THE SUMMER MEETING IN CHICAGO

The thirty-ninth summer meeting of the Society was held at the University of Chicago from Monday, June 19, to Saturday, June 24, in conjunction with meetings of the American Association for the Advancement of Science and the Mathematical Association of America. About five hundred persons attended the meeting, among whom were the following two hundred thirty-one members of the Society:

C. R. Adams, R. B. Adams, R. P. Agnew, A. A. Albert, E. S. Allen, R. B. Allen, N. L. Anderson, J. V. Atanasoff, F. L. Axen, R. W. Babcock, W. C. Babcock, R. W. Barnard, I. A. Barnett, Walter Bartky, W. D. Baten, H. L. Beard, E. F. Beckenbach, E. R. Beckwith, Theodore Bennett, B. A. Bernstein, H. R. Beveridge, G. D. Birkhoff, E. M. Blake, H. F. Blichfeldt, G. A. Bliss, H. F. Bohnenblust, J. W. Bower, C. F. Bowles, F. W. Boyce, H. R. Brahana, H. W. Brinkmann, W. E. Brooke, G. S. Bruton, C. T. Bumer, R. S. Burington, W. H. Bussey, W. D. Cairns, Helen Calkins, G. A. Campbell, J. W. Campbell, R. D. Carmichael, W. F. Cheney, R. V. Churchill, G. R. Clements, A. B. Coble, L. M. Coffin, L. W. Cohen, J. B. Coleman, J. T. Colpitts, E. G. H. Comfort, A. H. Copeland, Paul Cramer, A. R. Crathorne, D. R. Curtiss, E. H. Cutler, H. N. Davis, L. E. Dickson, L. L. Dines, B. F. Dostal, Arnold Dresden, W. L. Duren, R. L. Echols, W. E. Edington, M. C. Eide, E. B. Escott, H. P. Evans, H. S. Everett, F. J. Feinler, B. F. Finkel, C. H. Fischer, C. B. Fisher, A. H. Fox, T. C. Fry, M. G. Gaba, W. H. Garrett, H. H. Germond, Lachlan Gilchrist, B. P. Gill, D. C. Gillespie, Cornelius Gouwens, M. C. Graustein, W. C. Graustein, C. H. Graves, L. M. Graves, T. N. E. Greville, F. L. Griffin, B. L. Hagen, Harris Hancock, W. L. Hart, W. W. Hart, M. L. Hartung, O. C. Hazlett, E. R. Hedrick, M. R. Hestenes, E. H. C. Hildebrandt, T. H. Hildebrandt, F. H. Hodge, P. G. Hoel, T. F. Holgate, D. L. Holl, J. C. Hughes, E. V. Huntington, M. H. Ingraham, Dunham Jackson, Nathan Jacobson, R. D. James, C. M. Jensen, S. A. Joffe, Evan Johnson, M. M. Johnson, L. C. Karpinski, A. E. Kennelly, H. J. Kersten, J. M. Kinney, Alfred Korzybski, W. C. Krathwohl, W. D. Lambert, A. W. Landers, Jr., M. K. Landers, E. P. Lane, G. B. Lang, R. E. Langer, Lincoln LaPaz, G. A. Larew, C. G. Latimer, V. V. Latshaw, D. H. Lehmer, D. N. Lehmer, V. F. Lenzen, Tullio Levi-Civita, D. C. Lewis, F. P. Lewis, R. B. Lindsay, M. I. Logsdon, A. C. Lunn, Dorothy McCoy, E. A. McDougle, James McGiffert, W. D. MacMillan, J. D. Mancill, W. A. Manning, Morris Marden, William Markowitz, Anna Marm, R. G. Mason, A. E. May, J. R. Mayor, C. W. Mendel, W. I. Miller, E. C. Molina, E. I. Moody, C. N. Moore, T. W. Moore, E. M. Morenus, R. E. Moritz, C. B. Morrey, Jr., Richard Morris, T. A. Mossman, E. J. Moulton, C. O. Oakley, H. L. Olson, F. W. Owens, H. B. Owens, Gordon Pall, Z. M. Pirenian, E. C. Pixley, H. H. Pixley, Tibor Radó, W. R. Ransom, Nicholas Rashevsky, Francis Regan, W. T. Reid, C. E. Rhodes, R. G. D. Richardson,

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The chief feature of the meeting was the number of invited addresses by European and American mathematicians. Of these addresses, four, namely those by Professors G. D. Birkhoff, L. E. Dickson, and Lipót Fejér, and the first address by Professor Tullio Levi-Civita were an hour in length and each was a report on achievement in some field of mathematics. The other invited addresses were somewhat shorter. The Society is indebted to the Century of Progress for making possible the participation of Professor Lipót Fejér of Budapest and Professors Tulio Levi-Civita and Enrico Bompiani of Rome.

On Tuesday afternoon Professor Levi-Civita and Professor G. D. Birkhoff gave addresses on *Some mathematical aspects of the new mechanics*, and *Quantum mechanics and asymptotic series*, respectively. This was a joint meeting of the Society and Section A of the A.A.A.S. at which members of the American Physical Society were guests of the mathematicians. About five hundred attended this session. President Coble presided. Professor Levi Civita's address has appeared in the August issue of this Bulletin; Professor Birkhoff's address will appear in the October issue.

A second session of the Society and Section A was held on Wednesday morning at which Professor Fejér spoke on *The infinite sequences arising in the theories of harmonic analysis, of interpolation, and of mechanical quadratures*. This was followed by papers entitled *On the use of Cesàro means in determining criteria for Fourier's constants* by Professor C. N. Moore, and *Certain problems of closest approximation* by Professor Dunham Jackson. President Coble presided. Professor Fejér's address has appeared in the August issue of this Bulletin; the addresses

of Professors Moore and Jackson will appear in early issues of this Bulletin.

On Wednesday afternoon the Society and Section A held a symposium on geometry at which the following addresses were delivered: *Nets on a surface and extension of trigonometry* by Professor Levi-Civita, *Invariant methods in differential geometry* by Professor W. C. Graustein, and *Deformations of higher species of surfaces and manifolds* by Professor Bompiani. Professor C. N. Moore presided.

The Friday afternoon session was devoted to number theory. Professor L. E. Dickson spoke on *Recent progress in additive number theory*. This address, to be published in the October issue of this Bulletin, was followed by a number of contributed papers. President Coble presided.

The sessions on Monday afternoon, Thursday morning, and Friday morning were devoted to short contributed papers.

The Mathematical Association met on Tuesday morning and Thursday afternoon. On Saturday afternoon the Society met in a joint session with Section A and the Mathematical Association at Northwestern University where Professor T. F. Holgate gave an illustrated talk on *Mathematical reminiscences of the World's Fair of* 1893. Professor H. E. Slaught presided.

Following this final session, tea was served by the ladies of the Department of Mathematics of Northwestern University.

The meeting on Wednesday afternoon was held in the Italian Building of the Century of Progress Exposition. With the exception of the Saturday session at Northwestern University, all other sessions were held at the University of Chicago.

On Friday evening a joint dinner of the Society and Mathematical Association was held in Judson Court. Two hundred forty-four mathematicians and their guests attended this dinner. Professor G. A. Bliss acted as toastmaster and called upon Professor Fejér who extended his greeting to the Society in Hungarian, after which this was translated by Mr. Medgyesy, the Hungarian Consul General. Professor Levi-Civita spoke in Italian, and Professor Bompiani translated his talk. Then Professor Bompiani spoke. Professor Niels Bohr spoke on the situation of mathematicians and physicists in Central Europe. Talks by Presidents Arnold Dresden and A. B. Coble concluded the program of the evening.

On Monday evening members of the Society were guests of the Century of Progress at a reception given by the Century of Progress and the A.A.A.S. to the foreign guests.

Most of the visiting mathematicians were housed in Judson Court, residence hall for men of the University of Chicago. Meals were served here. This building and the Common Room in Eckhart Hall served as social centers for the meeting. The ladies of the Department of Mathematics of the University of Chicago served tea daily at Eckhart Hall. This added much to the pleasure of the meetings. The arrangements made by the local members for the meetings and for the comfort of the visitors were carried through with great success. In spite of a heavy program and the attractions of the Century of Progress Exposition, fine facilities for social contacts were afforded. The gratitude of the Society to those making these arrangements was voiced by Professor C. R. Adams at the meeting on Friday when a rising vote of thanks to the local committee and others was unanimously passed.

The Council met on the evening of Tuesday, June 22.

The following fourteen persons were elected to membership in the Society:

Mrs. Jean Fleming Brown, Hunter College High School, New York City;
Professor Herman Kyle Fulmer, Georgia School of Technology;
Mr. Norman L. Haight, Sperry Gyroscope Company, Brooklyn;
Professor Paul Laurence Hasty, Wilberforce University;
Mr. Junius John Hayes, University of Utah;
Miss Ethel Marie Hove, Wayne State Teachers College;
Miss Mary Catherine Strain, Park College;
Dr. M. O. Thomas, Annamalai University;
Miss Margaret Elizabeth Waldron, Rhode Island College of Education;
Mr. James Fletcher Wardwell, Johns Hopkins University;
Mr. Berthold R. Wicker, St. Mary's College, Winona, Minnesota;

As nominees of Yale University:

Mr. Howard Carl Frantz; Mr. Saunders MacLane; Mr. John Cochran Montgomery.

The titles and cross references to abstracts of the papers read at the sessions follow below. The papers were read as follows: papers numbered 1 to 15 on Monday afternoon, Professor G. D. Birkhoff presiding; papers numbered 16 to 32 on Thursday morning, Professor W. C. Graustein presiding; papers num-

bered 33 to 54 on Friday morning, Professor E. B. Stouffer presiding; and papers numbered 55 to 58 on Friday afternoon, President A. B. Coble presiding. Miss Newton was introduced by Professor E. P. Lane, Mr. Peek by Dr. T. C. Fry, Mr. Lyle and Mr. Sharpe by Professor H. M. Gehman, Professor Creedy by Professor Tomlinson Fort, Mr. Alden by Professor F. R. Bamforth, and Mr. Oberg by Professor Dunham Jackson. The paper of the late Professor Kellogg was transmitted by Dr. J. J. Gergen, and that of the late Dr. Paley by Professor J. D. Tamarkin. Papers whose abstract numbers are followed by *t* were read by title.

1. *Non-commutative polynomials and cyclic algebras*, by Mr. Nathan Jacobson. (Abstract No. 39–5–191.)

2. *Fundamental regions for the simple group of order* 168 *in* S_4, by Dr. W. I. Miller. (Abstract No. 39–7–192.)

3. *On algebraic rings and Abelian groups*, by Professor H. S. Vandiver. (Abstract No. 39–7–193–*t*.)

4. *Degree and class of multiply transitive groups*, by Professor W. A. Manning. (Abstract No. 39–3–113.)

5. *Every large number is a sum of nine values of a cubic polynomial in* x, by Professor L. E. Dickson. (Abstract No. 39–7–194.)

6. *A set of four postulates for Boolean algebra in terms of the "implicative" operation*, by Professor B. A. Bernstein. (Abstract No. 39–7–195.)

7. *A system of independent mathematical postulates for Dirac's quantum mechanics*, by Mr. A. M. Tuttle. (Abstract No. 39–7–196.)

8. *The problem of Lagrange with finite side conditions*, by Dr. J. W. Bower. (Abstract No. 39–7–237.)

9. *Summary of results and proofs on Fermat's last theorem* (seventh paper), by Professor H. S. Vandiver. (Abstract No. 39–7–197–*t*.)

10. *Integral domains of rational generalized quaternion algebras*, by Professor A. A. Albert. (Abstract No. 39–7–198–*t*.)

11. *A second set of independent postulates for the informal Principia Mathematica*, by Professor E. V. Huntington. (Abstract No. 39–7–235–*t*.)

12. *Postulates for logic*, by Mr. T. P. Palmer. (Abstract No. 39–7–232–*t*.)

13. *Groups in which every operator has at most a prime number of conjugates*, by Professor G. A. Miller. (Abstract No. 39–7–233–*t*.)

14. *Positive quaternary quadratic forms representing all but a finite number of integers*, by Dr. A. E. Ross (National Research Fellow). (Abstract No. 39–7–228–*t*.)

15. *The sum of like powers of a series of numbers forming an arithmetical progression and the Bernoulli numbers*, by Professor I. J. Schwatt. (Abstract No. 39–7–236–*t*.)

16. *Bilinear correspondences in two and in three dimensions*, by Professor H. L. Olson. (Abstract No. 39–7–199.)

17. *Consecutive covariant configurations at a point of a space curve*, by Miss A. V. Newton. (Abstract No. 39–7–200.)

18. *Consistency of the conditions determining Kollektivs*, by Professor A. H. Copeland. (Abstract No. 39–7–201.)

19. *Invariance of the property of admissibility under certain general types of transformations*, by Mr. T. N. E. Greville. (Abstract No. 39–7–202–*t*.)

20. *The application of the theory of admissible numbers to time series with variable probability*, by Dr. Francis Regan. (Abstract No. 39–7–203.)

21. *Some new theorems on limits of variation*, by Mr. R. L. Peek, Jr. (Abstract No. 39–5–183.)

22. *The probability law for the sum of n independent variables when each is subject to the law* $(1/(2h))\operatorname{sech}(\pi x/(2h))$, by Professor W. D. Baten. (Abstract No. 39–5–184.)

23. *On Finsler spaces*, by Professor M. S. Knebelman. (Abstract No. 39–5–186–*t*.)

24. *A canonical form for a set of vectors*, by Professor M. S. Knebelman. (Abstract No. 39–5–185–*t*.)

25. *A neighborhood treatment of general topological spaces*, by Mr. W. O. Gordon. (Abstract No. 39–5–187–*t*.)

26. *Points of connectivity*, by Professor H. M. Gehman. (Abstract No. 39–7–204–*t*.)

27. *Some theorems on triodic continua*, by Dr. N. E. Rutt. (Abstract No. 39–7–205–*t*.)

28. *A type of homogeneity for continuous curves*, by Mr. C. H. Wheeler, III. (Abstract No. 39–7–206–*t*.)

29. *Centers and axes of symmetry*, by Mr. R. R. Lyle. (Abstract No. 39–7–207–*t*.)

30. *Arcs of symmetry*, by Mr. B. B. Sharpe. (Abstract No. 39–7–208–*t*.)

31. *Note on Riemannian function space geometry*, by Dr. H. P. Thielman. (Abstract No. 39–7–227–*t*.)

32. *The completely symmetric rational self-dual septimic*, by Dr. D. C. Duncan. (Abstract No. 39–7–230–*t*.)

33. *The clock problem in relativity* (second paper), by Professor J. W. Campbell. (Abstract No. 39–7–209.)

34. *A hyperbolic vector diagram and symbolic method*, by Professor Frederick Creedy. (Abstract No. 39–7–210.)

35. *Certain problems in the theory of closest approximation*, by Dr. P. G. Hoel. (Abstract No. 39–7–211.)

36. *Discontinuous solutions of the non-parametric problem of Mayer in the calculus of variations*, by Dr. W. T. Reid. (Abstract No. 39–7–212.)

37. *Sufficient conditions for the problem of Bolza in the calculus of variations*, by Dr. M. R. Hestenes. (Abstract No. 39–7–213.)

38. *Asymptotic solutions of certain ordinary differential equations associated with multiple roots of the characteristic equation* (preliminary report), by Mr. H. L. Turrittin. (Abstract No. 39–7–214.)

39. *Functional invariants of integro-differential equations*, by Professor I. A. Barnett. (Abstract No. 39–7–215.)

40. *Cauchy's problem for systems of second order partial differential equations*, by Dr. E. W. Titt (National Research Fellow). (Abstract No. 39–7–216.)

41. *On the location of the critical points of Green's function for a plane region*, by Professor J. L. Walsh. (Abstract No. 39–5–182–*t*.)

42. *On the element of decomposition of a doubly periodic function of the second kind*, by Professor M. A. Basoco. (Abstract No. 39–5–189–*t*.)

43. *On the arithmetized Fourier developments of certain doubly periodic functions of the second kind*, by Mr. G. D. Nichols. (Abstract No. 39–5–190–*t*.)

44. *Properties of differential equations arising from properties of the solution curves*, by Mr. H. H. Alden. (Abstract No. 39–7–217–*t*.)

45. *Converses of Gauss' theorem on the arithmetic mean*, by Professor O. D. Kellogg. (Abstract No. 39–7–218–*t*.)

46. *Derivatives, difference quotients, and Taylor's formula*, by Dr. Hassler Whitney (National Research Fellow). (Abstract No. 39–7–219–*t*.)

47. *Systems of algebraic difference equations*, by Professor J. F. Ritt and Dr. J. L. Doob (National Research Fellow). (Abstract No. 39–7–220–*t*.)

48. *The general case of non-homogeneous linear differential equations*, by Dr. W. J. Trjitzinsky. (Abstract No. 39–7–221–*t*.)

49. *On mechanical quadratures*, by Professor J. A. Shohat and Dr. Clement Winston. (Abstract No. 39–7–222–*t*.)

50. *A note on the Jacobi condition for parametric problems in the calculus of variations*, by Dr. M. R. Hestenes. (Abstract No. 39–7–223–*t*.)

51. *A special integral function*, by Dr. R. E. A. C. Paley. (Abstract No. 39–7–231–*t*.)

52. *The approximate solution of integral equations*, by Mr. E. N. Oberg. (Abstract No. 39–7–238–*t*.)

53. *The generalized Bernoulli polynomial and its relation to the Riemann zeta function*, by Dr. B. F. Kimball. (Abstract No. 39–7–229–*t*.)

54. *Concerning the equilibrium point of Green's function for an annulus*, by Professor A. J. Maria. (Abstract No. 39–7–234–*t*.)

55. *Franklin magic squares*, by Professor D. N. Lehmer. (Abstract No. 39–7–224.)

56. *A generalization of a theorem of Dirichlet*, by Professor A. A. Albert. (Abstract No. 39–7–225.)

57. *Note on the invariants of the class group of a cyclic field*, by Professor C. G. Latimer. (Abstract No. 39–7–226.)

58. *On imaginary quadratic fields whose class number is unity*, by Dr. D. H. Lehmer. (Abstract No. 39–5–188.)

M. H. INGRAHAM, *Associate Secretary*

THESES ON CALCULUS OF VARIATIONS, 1931–32

Contributions to the Calculus of Variations, 1931–1932. Theses submitted to the Department of Mathematics at the University of Chicago. University of Chicago Press, February, 1933. 523 pp.

The present volume, uniform in appearance with the one published under similar title in 1930 (reviewed in this Bulletin, vol. 38 (1932), p. 617), contains eight doctoral dissertations, one master's thesis (No. 10 below), and an article (No. 6 below), written in pursuit of unalloyed scientific interest, rather than "submitted in candidacy for" a degree. It is gratifying to observe that the reception accorded the frankly experimental "contributions" of 1930 was sufficiently enthusiastic to justify a continuation of the venture. In size, in proof reading, in general appearance, this volume surpasses its predecessor; in significance of content it also marks an advance. In the preface, signed by Professors Bliss and Graves, under whose direction the theses contained in the volume were written, it is pointed out that one of the advantages of this form of publication of doctoral theses is that it makes it possible to give "the candidate for the Ph.D. degree as much freedom in his writing as is consistent with mathematical accuracy and clearness of style, even if the result is more verbose than could reasonably be approved for publication in a journal." It is therefore to be understood that the volume has not been edited, that each author is responsible for his own paper only, and that the book is to be looked upon as a collection of individual papers, linked together only in so far as their subjects are related, and by the incidental fact that they all represent work done by the Department of Mathematics of the University of Chicago.

The book consists of ten parts:

1. *Edge conditions for multiple integrals in the calculus of variations*, by J. E. Powell, (pp. 1–62).

2. *The Euler-Lagrange multiplier rule for double integrals*, by M. Coral, (pp. 63–94).

3. *The condition of Mayer for discontinuous solutions of the Lagrange problem*, by R. A. Hefner, (pp. 95–130).

4. *A problem in the calculus of variations suggested by a problem in economics*, by H. H. Pixley, (pp. 131–190).

5. *Functions of lines and the calculus of variations*, by R. G. Sanger, (pp. 191–294).

6. *Sufficient conditions for a problem of Mayer in the calculus of variations*, by G. A. Bliss and M. R. Hesteness, (pp. 295–338).

7. *Sufficient conditions for the general problem of Mayer with variable endpoints*, by M. R. Hestenes, (pp. 339–360).

8. *The problem of Bolza and its accessory boundary value problem*, by Kuen-Sen Hu, (pp. 361–444).

9. *Jacobi's condition for multiple integral problems of the calculus of variations*, by A. W. Raab, (pp. 445–474).

10. *A history of the classical isoperimetric problem*, by T. I. Porter, (pp. 475–523).

To begin with an account of the historical papers (Nos. 5 and 10), they review the literature on the subjects mentioned in their titles in a coherent, but not a critical, way. Dr. Sanger presents in two successive chapters the definitions for the derivative and the differential of a functional, as given by Volterra and Fréchet, respectively, and as developed and modified by various later writers; the third chapter deals with the applications of these concepts to the calculus of variations; it is followed by a bibliography consisting of eighty-three titles. No sharp distinction is made between what, using E. H. Moore's terminology, might be called functions on C to A, that is, functionals, which associate a real number with every function of a certain class, and functions on C_1 to C_2, that is, functional operations by means of which a function is made to correspond to a function. The work of Mandelbrojt, for instance, presented on pages 238–239 belongs in the latter field; most of the other work to the former. There is ample room for differences of opinion as to the relative importance to be attached to the various contributions dealt with in a historical survey of this character; it is not easy to obtain an adequate perspective over a field of which a great deal is still in the early stages of development and of which the importance can perhaps be appreciated only after the lapse of years.

The subject treated by Mr. Porter, more restricted in character, and one which has received complete and detailed study, lends itself more readily to a critical discussion. The problem with which nearly the entire paper is concerned is that of proving that among all closed plane curves of given length the circle encloses maximum area. An interesting list of seventy-five titles closes the paper. That the literature relating to the extensions and generalizations of this problem is still growing is shown for example by Abstract 114, published on page 208 of volume 39 of this Bulletin.

Rather apart from the rest of the volume in character and interest is paper No. 4, concerned primarily with conditions for a maximum of the integrals $\int f(x, y, y')\,dx$ and $\int f(x, y, y', y'')\,dx$, in the first of which y and y' are allowed to possess a finite number of finite discontinuities, while in the second y is supposed to be continuous, but y' and y'' may have a finite number of finite discontinuities. The conclusions reached by Dr. Pixley are that, in general, problems of this character, under his unrestricted conditions of admissibility, do not possess a solution. It would therefore be of special interest to establish a connection between this dissertation and the paper of Razmadzé (Mathematische Annalen, vol. 94 (1925), p. 1) in which an analogous problem with different conditions and different conclusions is discussed.

In an introductory section certain problems in economics are presented which lead to such problems of the calculus of variations as are treated in this paper. It must be confessed that these questions of the calculus of variations are of quite sufficient interest in themselves, and that the importance of the monopoly problems to which they are to be applied escapes the reviewer. It is not clear what is the economic significance of the function $2f$ introduced on page 185, or of the conditions imposed on the constants in this function in order to obtain a non-vacuous result. It is to be hoped that the conclusions of this paper are not to be interpreted as an indication that most problems in economics when formulated rigorously fail to possess solutions.

Of the seven remaining parts, three (Nos. 1, 2, 9) are concerned with multiple

integrals and four (Nos. 3, 6, 7, 8), with the problems of Lagrange, Mayer, and Bolza. We shall briefly discuss each of these groups.

Dr. Powell considers multiple integrals of the form $\int_S f(x, y, p)\, dx$ in which x and y are respectively n- and m-partite variables, and p is the set of partial derivatives of y with respect to x. The admissible manifolds are those which are defined by a set of functions $y(x)$ continuous over a bounded, closed, connected region S of x-space, which consists of a finite number of cells in which the partial derivatives of y are continuous. The vanishing of the first variation leads to the Euler-Lagrange equations, and to a generalization of the Erdmann-Weierstrass corner condition along the edges of the manifold, that is, the curves corresponding to the boundaries of the cells. Analogous *edge conditions* are then obtained for the related isoperimetric problem, in which a fixed value is assigned to a second integral of the same type as the one which is to be minimized. After a brief section dealing with a discontinuous integrand f, the edge condition is obtained once more by means of an extension to multiple integrals of Mason's lemma for double integrals under slightly more general hypotheses.

The extension of the Euler-Lagrange multiplier rule to problems involving multiple integrals and partial differential equations has progressed very slowly. A paper by Gross (1916) appears to be the only one in which the problem was materially advanced. In his contribution to the volume under review, Dr. Coral carries the work a considerable distance forward. The first chapter gives a simple treatment of the restricted problem formerly treated by Gross; the second chapter applies the same method to the solution of the problem of minimizing the double integral $\iint f(x, y, z_i, p_i, q_i)\, dx\, dy$, $(i=1, 2)$, by means of functions of a certain class which are moreover required to satisfy the differential equation $\phi(x, y, z_i, p_i, q_i)=0$. The new method developed by the author consists in reducing the given problem to an associated Lagrange problem involving only simple integrals.

In the last paper of this group, Dr. Raab applies Bliss' method for the treatment of second-order conditions to a multiple integral of the same form as the one for which edge conditions were derived by Dr. Powell. In this way he obtains as the analog of Jacobi's equation for the simple integral a system of linear partial differential equations of the second order of elliptic type. As a first step towards obtaining an analog of Jacobi's condition for this problem a result is obtained which, in the main, amounts to the following: If for a minimizing manifold which satisfies Legendre's condition in the stronger form, the Jacobi system of equations possesses a solution of class C'' which vanishes along a Green's manifold M within the domain of integration, then the corresponding first partial derivatives must all vanish on M. A Green's manifold is understood to be a closed $(n-1)$-dimensional manifold which bounds a portion of x-space, and for which the extension of Green's lemma is valid. A corresponding result is obtained for the multiple integral in parametric form.

Of the last group of papers in this volume, two have appeared in volume 35 of the Transactions of this Society, namely, No. 6 (pp. 305–327) and No. 7 (pp. 479–491). A discussion of their contents is therefore not in order here; suffice it to say that they complete and simplify in an important respect the recent work done on the problems of Mayer and Bolza by Morse and Bliss.

Paper No. 3 extends to the Lagrange problem in parametric form the results obtained by Graves (American Journal of Mathematics, vol. 52 (1930)) for discontinuous solutions of problems in n-space. Dr. Hefner's paper follows closely not only the methods used by Graves, but also the language used by that author. The most important result obtained is the extension of the Mayer condition to extremaloids, that is, to sets of functions $y_i(x)$, $\lambda\alpha(x)$, of which the latter are continuous except possibly at corners, the former continuous, while together they satisfy the Euler equations and the corner conditions. The definition of *conjugate points* introduced by Graves is carried over to this problem and it is shown that these points are obtained for values of the parameter which cause a certain determinant to vanish or to change sign.

In paper No. 8, Dr. Hu deals with the problem of Bolza in essentially the same form as was used by Morse (American Journal of Mathematics, vol. 53 (1931)). He gives, however, a new formulation of the *accessory problem* in terms of a function $I_2(\xi, \eta)$ involving the end values of the variations as well as an integral connected with the second variation, a system of linear differential equations, end conditions, and the additional condition

$$\int_{x_1}^{x_2}(\xi_1^2+\xi_2^2+\eta_i\eta_i)dx=1.$$

This accessory problem is shown to be equivalent to a boundary value problem involving a parameter σ. It is in terms of this boundary value problem that the principal results of the paper are obtained, namely, a necessary and sufficient condition for a permanent sign of $I_2(\xi, \eta)$ for all variations which satisfy the conditions of the accessory problem, and a set of sufficient conditions. Further study of the boundary value problem and the related minimum problem lead to oscillation and comparison theorems, stated in terms of the distribution of the focal points of the manifold determined by the end conditions. This paper represents a valuable contribution to the study of the interrelations between the field of boundary value problems and the calculus of variations.

ARNOLD DRESDEN

HANCOCK ON ALGEBRAIC NUMBERS

Foundations of the Theory of Algebraic Numbers. By H. Hancock. Vol. I. *Introduction to the General Theory.* 27+602 pp. Vol. II. *The General Theory.* 26+654 pp. New York, Macmillan, 1931–32.

Hancock's new book is easily the most voluminous treatise in the literature on algebraic numbers and ideal theory. The publication of such a handbook in English is in itself quite remarkable since ideal theory, both in algebraic fields and in abstract rings, mainly has been a continental or more specifically a German branch of mathematics. Textbooks in other languages have been few. Reid's book containing an elementary treatment of quadratic fields seems to have been the only contributions on the subject in English.

Since the time of Gauss the theory of algebraic numbers has been considered as one of the spires of mathematics and Hancock approaches his subject in an appropriate spirit: "By virtue of the simplicity of its foundations and the rigor of its deductions, arithmetic stands alone in the beauty and harmony of its truths. A divine gift, it offers proof that the mind is a reality attested by the sciences on the one hand, and the philosophies on the other. The province of arithmetic in this high position between science and philosophy, is both to serve and to be served in the quest of higher truths."

In the introduction the author discusses the natural problem of the foundations of ideal theory and decides to follow mainly the ideas of Dedekind, while their relations to the theories of Kronecker and Hensel are emphasized. This seems to be the simplest and most practical way of building up the ideal theory in algebraic fields. The relation between the various theories of Kummer, Dedekind, Zolotareff, Kronecker, Hensel, Prüfer, and others are fairly well known at the present time and a parallel development of all theories would be superfluous. Dedekind's point of view is in many ways both the simplest and the most general and this opinion, it seems to me, is confirmed by the facility with which the theory can be extended to abstract rings. It is true, however, that Hensel's theory has received a new impetus through the recent investigations of Krull on rings with generalized absolute values (Bewertungsringe).

Hancock's book deals exclusively with the classical ideal theory in algebraic fields. In the first volume one finds the foundations, beginning with the principal properties of polynomials, resultants, discriminants, and other preliminary notions. Then the definitions and main properties of fields, algebraic integers, units, bases, discriminants, and applications to quadratic and cubic fields are given, all following the well established lines of the theory. In the Chapters 5–7 one obtains a detailed discussion of the algebraic moduli introduced by Dedekind and in a form which is almost identical to the presentation in the supplements of the fourth edition of Dirichlet's *Number Theory.* The following Chapter 8 contains the elements of Kronecker's modular systems. In my opinion it would have been wiser to omit these chapters on modular theory in volume I and include them at the beginning of volume II where they are applied for the first time. Volume I would then have contained only

the elementary parts of ideal theory. As it stands the modular theory forms an unnecessary obstacle to the ensuing chapters of the first volume.

The remaining half of the first volume deals with ideal theory in quadratic and cubic fields. It gives a comprehensive account of the elements of ideal theory and the clear and easy style should make it a good introduction for beginners in the subject. The presentation seems to be inspired by Sommer's book on algebraic numbers, since both the subject matter and the treatment show great similarities in both books. Chapter 9 treats the properties of ideals in quadratic fields and next follows a discussion of the quadratic law of reciprocity, the norm residue symbol of Hilbert, and the theorems on genera in quadratic fields. There is one chapter containing applications to Fermat's theorem for the exponents $n=3$ and $n=4$, another on the relation between quadratic forms and ideals, and a final chapter on cubic fields. The treatment of quadratic fields is a little cumbersome due to the necessity of splitting each proof into cases. This holds not only for Hancock's book but also for most other accounts. At the present time it should be possible to condense this theory by using the same methods as for general relative cyclic fields.

I should like to end this discussion of the first volume by an observation in connection with the determination of the prime ideal decomposition of a prime $p \neq 2$ in a quadratic field $K(m^{1/2})$. When $m \not\equiv 0 \pmod{p}$ the decomposition depends on the value of the symbol (m/p), that is, on the solvability of a congruence $x^2 \equiv m \pmod{p}$. Both in Sommer and Hancock it is assumed that an eventual solution a is taken such that $a^2 - m \not\equiv 0 \pmod{p^2}$ and then

$$p = \mathfrak{p}_1\mathfrak{p}_2 = (p,\ m^{1/2} - a)(p,\ m^{1/2} + a).$$

It should be observed that this condition on a is unnecessary and that it could have been avoided by a slightly different proof.

The second volume contains a large number of topics from the theory of ideals. Hancock deduces the principal properties of ideals using the full apparatus of modular theory and following closely the presentation by Dedekind. This is certainly not the simplest method of introducing ideal theory, but on the other hand, it gives a more complete background for the results. However, I should have preferred to see the theory developed along the more modern lines of abstract ideal theory, giving, for instance, the much shorter proof by Krull for the fundamental theorem.

In the first two chapters the main properties of moduli, ideals, integral ideals, norms, and residue systems are discussed and then follows a chapter on prime ideals including the proof for the unique decomposition theorem, various elementary properties of prime ideals, and also an introduction of ideal classes. Another chapter gives an outline of Kronecker's ideal theory and then there is a second proof of the main theorem by Hurwitz. The theory of discriminant divisors is given in the same way as in Bachmann's book on the arithmetic of algebraic numbers. Dirichlet's theory of units is developed in two different ways and Minkowski's geometrical considerations are applied to the representation of the class number by means of the Dedekind zeta-function and to Dedekind's formula for the class number of rings. There is a short account of the theory of decomposable forms and next an extensive discussion of relative fields with applications to relative quadratic fields and with some elementary

remarks on class fields. The results of Dedekind and Hilbert on the connection between ideals and group theory are derived, preceded by an account of the Galois theory of almost 100 pages. The final chapter contains some of the properties of p-adic numbers.

As a whole the second volume is not as well composed as the first. The various topics stand isolated with little or no attempt to connect them to an organic structure. The long elementary discussion of the Galois theory might well have been omitted since a book on algebraic numbers cannot be expected to be a textbook on all algebraic theories. Some of the topics are rather obsolete: as an example I mention the theory of common index divisors which, according to more recent points of view, owe their importance to the imperfection of the method applied and not to their intrinsic properties.

I feel certain that Hancock's book will be used as textbook in a large number of courses on algebraic numbers and that it will prove very helpful, particularly for those students whose knowledge of the German language does not permit them to read the sources with facility. As I have already mentioned, the book is clearly written and several of my graduate students in a course on ideal theory have expressed their satisfaction with its presentation.

However, it cannot be denied that the book has various defects, some of which I consider to be serious. First of all, it is too bulky. I have already indicated several chapters which could have been omitted without great loss But there are other reasons for the great size of the book than the richness of the material: almost every theorem of importance is proved in at least two ways and several in still more ways. The fundamental theorem on the unique decomposition of ideals is proved three or four times in Volume 2 and in addition the full ideal theory is developed independently both for quadratic and cubic fields in Volume 1. In some cases it may clarify a theory to give different proofs, but to this extent it seems an unnecessary repetition only tiring the reader.

The main sources for Hancock's book are obviously: Sommer's *Lectures on Number Theory*; Bachmann's *General Arithmetic of Number Fields*; Dedekind's Supplements to Dirichlet's *Number Theory*; Hilbert's *Report*; and Weber's *Algebra*. These are all admirable contributions to the theory of ideals, but it should be remembered that they were published more than 25 years ago and most of them are considerably older. Since then the theory of algebraic numbers and ideals has made great strides, with new progress in older problems and new channels opened up. Of this development Hancock's book gives no indication. It leaves the student without bringing him in touch with modern problems and without bridging the gap to present day periodical literature.

Hancock's book will probably be widely used as a reference book and hence have an influence on future English nomenclature. This prompts me to make a final observation about the notation used in the book. It seems to me that the translations of the German terms are not always fortunate, although I admit that it may be a matter of taste. I should have preferred the term field to realm, fundamental field to stock realm, discriminant to basal invariant, conductor to ring leader, etc. Terms like Gattung and genuses should have been avoided.

OYSTEIN ORE

SHORTER NOTICES

Puzzles and Curious Problems, By H. E. Dudeney. London, Thos. Nelson and Sons, 1932. 195 pp.

Alte und Neue Mathematische Spiele. Eine Einführung in die Unterhaltungsmathematik. By G. Kowalewski. Leipzig and Berlin, Teubner, 1930. vi+145 pp.

Dudeney died in 1930 and the first of the volumes under review was prepared, from papers he had left behind, by his wife, the well known English novelist. A biographical sketch is given in the preface. For many years he was the famous inventor of mathematical recreations for Tit-Bits, Weekly Dispatch, Daily News, and the Strand Magazine, and he published four attractively written and ingenious volumes: *The Canterbury Puzzles* (1907; second ed., 1909), *Amusements in Mathematics* (1917), *Modern Puzzles and How to Solve Them* (1926), and *The World's Best Word Puzzles* (1926). Dudeney is also the author of the article on magic squares in the last edition of the Encyclopaedia Britannica.

In the present volume there are 365 problems classified under the headings: arithmetic and algebraic problems, geometric problems, moving counter problems, unicursal and route problems, combination and group problems, magic square, measuring, weighing, and packing problems, etc. *Solutions* occupy pages 127–189. There is a good index.

The work of Kowalewski is something of a very different character, and is filled with material of strong appeal for the mathematician. The motto for the book, quoted from a letter of Goethe to Schiller, is highly appropriate. It is: "Übrigens ist nur alles verhasst, was mich bloss belehrt, ohne meine Tätigkeit zu vermehren oder zu beleben." Kowalewski's single previous publication in this field was his pamphlet on the *Boss Puzzle und Verwandte Spiele* (Leipzig, 1921)*, but while at the University of Prague (1912–19), he gave lectures on mathematical recreations. In America the Boss puzzle is familiar as the fifteen puzzle, through papers of Johnson, Story, and Frazer; a detailed historical account of the problem is given in the monumental work of Ahrens (*Mathematische Unterhaltungen und Spiele*, 2d ed., Leipzig, vol. 2, 1918, Chapter 19).

Kowalewski's little book contains five chapters, the first (p. 1–20) of which is entitled *The colored cubes of Major MacMahon*. Here, as elsewhere, there is scarcely any reference to the literature of the subject, no hint as to where MacMahon discussed the problem. Hence it may be noted that the Major's paper *On the thirty cubes constructed with six coloured squares* was published in the Proceedings of the London Mathematical Society ((1), vol. 24 (1893), pp. 145–155). The statement of the problem, involving rotations of cubes, would take too much space. It is merely to be noted that two solutions of MacMahon's problem are set forth, a notable relation between them is indi-

* *Mathematica Delectans. Ausgewählte Kapitel aus der Mathematik der Spiele in gemeinverständlicher Darstellung*, Heft 1.

cated, a new game with MacMahon cubes is formulated and solved, and a new form of the cube game, involving hemispheres, is given.

The second chapter (p. 21–35), entitled *The mysterious grid or the thought reader*, seems to be a new mathematical game involving the binary scale of notation.

In the third chapter (pp. 35–61) on *The game of nim or fantan or three heaps*, it is noted that the mathematical theory of the game, and its name nim, were first given by C. L. Bouton, of Harvard University, in Annals of Mathematics, 1901, and it is remarked that "die weitgehendste Verallgemeinerung des Nimspieles verdanken wir dem berühmten amerikanischen Mathematiker E. H. Moore." This was published in the Annals, 1910. In both of these discussions the binary scale is employed. Most of the chapter is taken up with the discussion of modifications of the conditions of the game and with consideration of the relation of the discussion to that of Bouton and Moore.

The long fourth chapter (pp. 61–125) on *Puzzles of arrangement*, deals with a variety of problems, such as the Boss-puzzle, Sir Wm. R. Hamilton's dodecagon game (Quarterly Journal of Mathematics, vol. 5 (1862)), and a new kind of closed knight's move. The last chapter sets forth different forms of solitaire playing.

This valuable book notably supplements such works in English as those of Ball and MacMahon, and lives up to its motto throughout. Moreover the style of presentation is, for the most part, very pleasantly informal.

R. C. ARCHIBALD

Einführung in die theoretische Physik. In three volumes. By Clemens Schaefer. Volume 3, Part 1: *Elektrodynamik und Optik.* Berlin and Leipzig, de Gruyter, 1932. 918 pp.

A perusal of Professor Schaefer's admirable account of the classical electrodynamics and optics is like a visit to an old friend who is familiar with the great achievements of the past and can describe many modern discoveries in well-chosen language.

The first three chapters deal with static electric and magnetic fields and steady electric currents. The account of electromagnetic theory begins in Chapter IV, which contains a formulation of the laws for non-stationary processes. This is followed by a study of quasi-stationary currents and of electromagnetic waves in insulators and conductors. There is an admirable account of the optics of perfectly transparent media in which the phenomena of reflection are beautifully illustrated by diagrams indicating the flow of energy in the two media for the cases of partial and total reflection.

The chapter on the optics of crystals, supplemented by one on geometrical optics and diffraction, provides a clear elementary introduction to crystal theory.

There is a good presentation of the theory of electrons and dispersion which contains a brief account of its recent application by Larmor, Appleton, Nichols, and others to explain the striking fact that the range of the long waves of wireless telegraphy has a marked minimum for a wave-length of 200 m. The thermodynamical theory of radiation is well presented and the chapter on relativity closes with a sketch of the general theory, which the author regards as not yet fully established by the astronomical tests.

H. BATEMAN

The Theory of Functions. By E. C. Titchmarsh. Oxford University Press, 1932. x+454 pp.

Assuming a foundation of elementary analysis, as provided by Hardy's *Course in Pure Mathematics* (Cambridge, Fifth Edition, 1928), this volume introduces its readers to a number of branches of the theory of functions. A comprehensive first chapter on infinite processes is followed by eight chapters on the theory of functions of a single complex variable and four quite independent chapters on functions of a real variable, which are included because of the conviction of the author that their contents form a necessary part of the equipment of a student of modern analysis. The exercises include many useful theorems and form an attractive feature of the work. An extensive bibliography is an aid to further study.

The discussion throughout reveals the author's wide acquaintance with recent literature and contains many improvements and refinements of exposition derived from that source. The treatment of analytic functions is based on the Cauchy integral theorem. An analytic function is any differentiable function. The term region is left undefined, except for the remark that the interior of a square or circle is a region, and the theory of Jordan curves and boundaries of regions is left aside with the remark that the subject belongs to analysis situs. The chapters on analytic functions, residues, and analytic continuation are followed by a chapter on the maximum-modulus theorem containing results credited to Schwarz, Vitalli, Montel, Hadamard, Carathéodory, and Phragmen-Lindelöf. Chapter 6 on conformal representation contains an attractive discussion of simple functions and their application to the problem of the representation of a region on a circle. The next three chapters discuss the properties of functions defined by power series and Dirichlet series. They contain an exposition of the beautiful and powerful theory of over-convergence and carry the student to the frontier of the subject of integral functions.

The chapters on real functions consider the measure of linear point sets, Lebesgue integrals, derivates, functions of bounded variation, change of variable in a definite integral, convergence in mean, and repeated integrals. The final chapter, on Fourier series, includes the Riesz-Fischer theorem together with discussions relating to the uniqueness of trigonometrical series and Fourier integrals.

In evaluating a work of this extent it is important to consider the omissions. The geometrical side of analytic function theory receives scant consideration. There is little on algebraic functions, no mention of groups of linear transformations and the classes of functions defined by differential equations. The chapters on real functions contain very little on the descriptive properties of sets of points. The topics transfinite arithmetic, classification of functions, Baire and Suslin sets, Borel measure, metric density, and the well known Weierstrassian result regarding the representation of a continuous function by a uniformly convergent sequence of polynomials are excluded. The author has failed to utilize the opportunity to apply Lebesgue integration to contour integrals and the summation of series to power series. The discussion of summation of series is very brief and does not suggest its importance to the student of analysis.

Thus the student is led deeply into some parts of the theory of functions and left ignorant of others of comparable importance. The concentration on functions of a single variable is a further disadvantage. Works of this type, valuable as they are, show by their condensations, omissions, and inevitable lack of balanced emphasis, the importance of collections of monographs each devoted to the exposition of a single field.

The author and publishers are to be congratulated on the presentation of a work so useful and attractive. The typography is unusually clear and restful to the eye. As a whole the book is timely and accurate. It is convenient to have in one compact volume demonstrations of so many fundamental propositions with indications of many more in exercises and citations. A brilliant scholarship and much teaching experience have been combined in an interesting and reliable introduction to a large part of the theory of functions.

E. W. CHITTENDEN

La Relativité: 1. *Cinématique de la Relativité.* By E. Bauer. 2. *La Dynamique Relativiste et l'Inertie de l'Énergie.* By F. Perrin. 3. *Conséquences de la Relativité dans le Développement de la Mécanique Ondulatoire.* By L. de Broglie. 4. *La Théorie Einsteinienne de la Gravitation; les Vérifications Expérimentales.* By G. Darmois. 5. *Le Parallélisme Absolu et la Théorie Unitaire du Champ.* By E. Cartan. 6. *La Relativité; Conclusion Generale.* By P. Langevin. Paris, Hermann, 1932.

This is a series of six monographs, issued separately as parts of the series Actualités Scientifiques et Industrielles, and forming a symposium on Relativity under the auspices of the Centre International de Synthèse and directed by P. Langevin. The treatment is quite elementary and in general non-mathematical. M. Bauer's part deduces in some 23 pages the Lorentz transformation and its elementary consequences. The deduction is unsatisfactory because it is assumed, p. 18, that because when $x^2+y^2+z^2-c^2t^2=0$, then $x'^2+y'^2+z'^2-c^2t'^2=0$, it follows that $x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2$. M. Perrin's part (19 pages) gives the elementary dynamical consequences of the Lorentz formulas (transverse and longitudinal mass, variation of mass with velocity, etc.). In the third part (14 pages) L. de Broglie shows the connection between the Lorentz formulas and the fundamental quantum relation $E=h\nu$, and how the relativity theory resolves the conflict between the wave and corpuscular theories of light (pressure on a reflecting mirror, relation between Fermat's principle and the principle of Maupertuis). Part 4 (30 pages), by G. Darmois, discusses Schwarzschild's centro-symmetric solution of the equations of general relativity in free space and its application to the classical verifications of the theory (secular advance of perihelion of the planets, deflection of a ray of light passing near the sun, spectrum shift to the red). Part 5, by E. Cartan, is to the reviewer the most interesting of all. In less than twenty pages, and without mathematical symbols, is developed a readable and thought provoking account of the theory of distant parallelism and of the significance of torsion as opposed to curvature of space. In part 6, Langevin gives, in some 15 pages, a general summary.

Taken as a whole the set of six monographs forms a useful résumé of the elementary parts of relativity theory.

F. D MURNAGHAN

The Physical Significance of the Quantum Theory. By F. A. Lindemann. Oxford, Clarendon Press, 1932. 148 pp.

Many books have been written recently with the object of describing the mathematical methods and rules for applying the quantum theory to various physical phenomena. But a physicist feels that there is something lacking. He wishes to know why these particular rules are adopted, what physical principles are back of them. To answer by saying that they lead to results confirmed by experiment is not wholly satisfying. This book of Lindemann's is an attempt to meet this need of physicists.

The author's thesis is that the failure of the classical theories lies in the inadequacy of our spatio-temporal descriptions. If we try, for example, to measure the distance between two objects by placing a measuring rod between them, we can never get an exact result, because the very operation of bringing the rod in contact with the bodies deforms them and it. The ideal rigid solid does not exist. And so it is with all measurements in physics; approximate results only can be obtained. So taking Heisenberg's principle of uncertainty as a starting point, Lindemann obtains in a very simple way the characteristic values of the energy for a simple rotator, the harmonic oscillator and the hydrogen atom. Unfortunately, the first two do not give exactly the same results as the wave-mechanics yields. And so it cannot be said that he has got to the real physical significance of the quantum theory. However, he has made an interesting attempt; a physicist will find much of interest in the book even if he cannot accept all the arguments. And such attempts as this, even if they do not reach their goal, are well worth while. The book is well written, but it does seem that a book published at Oxford should spell the name of the discoverer of the electron correctly.

E. P. ADAMS

Aufgabensammlung zu den gewöhnlichen und partiellen Differentialgleichungen. By G. Hoheisel. (Sammlung Göschen, No. 1059.) Berlin, Walter de Gruyter, 1933. 148 pp.

This book, in the convenient pocket size of the Sammlung Göschen, is intended to accompany two other texts in the same collection, *Gewöhnliche Differentialgleichungen*, No. 920, and *Partielle Differentialgleichungen*, No. 1003. References to the earlier volumes are made occasionally to amplify the theory, but this volume is almost complete in itself. For each topic considered the theory is summarized and illustrated by examples, then followed by a brief list of problems. At the end of the treatment of ordinary differential equations there is a collection of 117 miscellaneous problems, making a total of 203 in the first part. The second part is devoted to partial differential equations and contains 33 problems grouped under the different topics.

The author has not intended to include routine drill problems but has made his selection with the idea of helping the student to a better conception of the underlying theory. There is a brief solution of each problem, but it does not immediately follow the statement and the conscientious student may seek his own solution before accepting hints from the writer.

W. R. LONGLEY

Mécanique Quantique et Causalité. D'après M. Fermi par André George. Paris, Hermann, 1932. 18 pp.

The present booklet, which appeared as the fifth publication of the well known series *Exposés de Physique Théorique*, is an illustrative commentary on a memoir by Enrico Fermi (Nuovo Cimento, new series, vol. 7, 10, pp. 361–366).

Causality is interpreted as determination of future events. After a brief review of the status of classical causality, the author proves, in a very elegant manner, the equivalence of the two quantum mechanical methods of defining states: (a) in terms of instantaneous values of physical quantities, (b) by means of ψ-functions. He then shows that if a physical quantity is measurable with precision at a given time, the value of that quantity can be calculated definitely for any later time. This fact is illustrated by a detailed consideration of a simple special case: one-dimensional uniform motion.

The restrictions of the causality principle spring mainly from Heisenberg's theorem of related indeterminacies, which claims the impossibility of simultaneous measurements of canonically conjugate quantities. L. de Broglie adds parenthetically the observation that knowledge of the present value of a physical quantity permits of no conclusion regarding its value at a time prior to the measurement, since the physical system may not have been in a pure state. In the conclusion it is pointed out in what manner the recent ideas of Bohr, Landau, and Peierls impose further restrictions upon the conventional understanding of causality.

The discussion is lucid and convincing; it makes no pretense to solve the causality problem in its entire complexity, but adds a few interesting side lights.

HENRY MARGENAU

Notions de Mécanique Ondulatoire; les Méthodes d'Approximation. By L. Brillouin. Paris, Hermann, 1932. 34 pp.

This is No. 39 of the series Actualités Scientifiques et Industrielles and the first of a "sub-series" devoted to quantum theory under the general direction of L. Brillouin. The connection between the wave-equation and the Hamilton first-order equation by which solutions of the latter may be used to furnish approximations to solutions of the former is carefully developed. A problem in one degree of freedom involving the method of steepest descent (*méthode de col*) for the approximate evaluation of definite integrals is worked out in detail. An account is given of Schrödinger's method of perturbations, and a theory applicable to large perturbations is sketched; this involves infinite determinants, and the treatment is heuristic, no convergence questions being discussed. A short section is devoted to non-conservative systems and Dirac's application of the method of variation of constants. The treatment assumes familiarity with the methods of wave-mechanics and is very useful in giving concreteness to questions which often remain very abstract in more elaborate treatments.

F. D. MURNAGHAN

Hydrodynamics. Report of the Committee on Hydrodynamics, National Research Council: Hugh L. Dryden, Francis D. Murnaghan, H. Bateman. Washington, National Research Council, 1932. 634 pp.

When one thinks of hydrodynamics, Lamb's invaluable treatise nearly always comes first to the mind. But the hydrodynamics of Lamb is mainly that of an ideal fluid, and there is more emphasis placed on getting satisfactory mathematical solutions of the various problems considered than there is on getting solutions that correspond to the results of observation. As Lord Rayleigh once remarked, "It is little wonder that practical men should declare that theoretical hydrodynamics has nothing at all to do with real fluids." The development of mechanical flight in particular, and other applications of fluid motion, in recent years, have made the study of real fluids a matter of great importance. An actual need existed for a comprehensive survey of what has already been accomplished in modifying the classical treatment of hydrodynamics so as to bring theory closer to experiment. The National Research Council is to be congratulated on their selection of authors, who have so admirably performed this service.

The opening chapter of this Report is contributed by Dryden, who gives a general view of the need for such a work, discusses the principle of dynamical similarity, and shows the significance of the Reynolds number in all cases of the flow of real fluids.

The second chapter, by Murnaghan, is mainly concerned with the calculation of the forces on aero-foils obtained by conformal transformation from a circular cylinder moving in a perfect fluid with circulation. This is a subject which is only briefly touched upon by Lamb. The treatment has many points of novelty, and copious references are given to the original sources.

The remainder of the volume, about six-sevenths of the whole, is contributed by Bateman. First there is a chapter on the physical properties of a viscous fluid with an excellent discussion of the experimental methods of determining the coefficient of viscosity. Then follows the mathematical theory of the motion of viscous fluids, and an application to those problems which have thus far been brought within the scope of the theory. The boundary layer theory of Prandtl receives an adequate treatment. The subjects of turbulent flow, and the influence of compressibility on the motion of viscous fluids, complete the Report. It would serve no useful purpose to attempt to give here an account of all the topics that are discussed. The physical interest is always kept in view, and there is constant reference to experimental results. One must be amazed at the encylopaedic knowledge of the author and his industry in gathering together and assimilating such a wealth of material. There are nearly two thousand different names listed in the index. Of course by no means all of them are referred to in the text, but references are given to their writings.

Except for an account of the present state of the theory of shock waves, or waves of discontinuity, in compressible fluids, the subject of wave motion, which forms nearly half of Lamb's treatise, is not included in this Report.

This Report is an important contribution to the subject of hydrodynamics, and it will be found to be of the greatest value by all who are interested in the newer developments of the subject.

E. P. ADAMS

Reelle Funktionen. Erster Teil: *Punktfunktionen.* By Hans Hahn. Leipzig, Akademische Verlagsgesellschaft, 1932. xi+415 pp.

As is indicated in the preface, this is hardly a new edition of the earlier (1921) *Theorie der reellen Funktionen* by the same author. Instead it is an entirely new treatment of the subject which has been occasioned, as the author remarks, by the great developments in this field during the past decade. The first glance through the pages of this book suffices to convince one of the truth of this assertion. The analytic point of view has been abandoned entirely, and has been supplanted by one which is purely set-theoretic. The result is admirable. All demonstrations are concise and short with no wasted words or repetition of arguments, a fact which attests the excellent organization of the material and at the same time presents a convincing argument for the set-theoretic point of view.

The treatment is complete in that nothing is presupposed on the part of the reader beyond a certain maturity of thinking capacity. The first chapter comprises a logical foundation for the notions of function and set and treats fundamental questions concerning systems of sets, cardinal numbers, and ordered sets. This is followed by a chapter devoted to point sets and including a treatment of abstract topological spaces, separable spaces, and complete spaces. Chapter 3 deals with the notion of continuity and, in the reviewer's opinion, contains one of the most interesting features of the book—a treatment of the most general type of transformation, not even necessarily single-valued. The author introduces the notion of upper and lower semi-continuity for such transformations and obtains substantial results without further restriction. This is followed by a natural definition of continuity and then, step by step, the author develops the properties of single-valued transformations, homeomorphisms, continuous functions, discontinuous functions, and function sets. The last two chapters are devoted to Borel sets and Baire functions, and to analytical sets, respectively; and they comprise, in addition to the perhaps better known results of Baire and Borel, also many of the more recent contributions from a host of authors including Suslin, Lusin, Sierpinski, Mazurkiewicz.

While hardly suitable as a textbook for the beginning class in real functions in most American universities, this book could be used to good advantage in a more advanced one. It is admirably suited, however, to the needs of the mathematical reader wishing to familiarize himself with the very recent developments in the field of real functions, since it brings him right up to date, including references to many articles as late as 1931.

G. T. WHYBURN

Probabilités et Morphologie. By A. Sainte-Laguë. Paris, Hermann, 1932. 30 pp.

This is a brief account of the elements of mathematical statistics (the probability curve, etc.) and of their application to such problems as the estimation of the racial purity of a given population. The treatment is entirely elementary. There are some interesting remarks on the inferences that can be drawn from *simultaneous* observations on a large population regarding the variation in *time* of a single individual.

F. D. MURNAGHAN

Geschichte der Elementar-Mathematik in systematischer Darstellung mit besonderer Berücksichtigung der Fachwörter. By J. Tropfke. Dritte verbesserte und vermehrte Auflage. Vol. I: *Rechnen*, vi+222 pp. 1930. Vol. II: *Allgemeine Arithmetik*, iv+266 pp. 1933. Berlin and Leipzig, Walter de Gruyter.

The earlier editions of this work have been exceedingly useful, not only for the great amount of material which they contain but also for their many references both to the sources and to secondary material. The first two volumes of the third edition promise well for those still to come, for the bulk of each has been considerably increased, and the number of foot notes has been enlarged in even greater proportion. The additions to the two volumes indicate our increasing knowledge of the mathematics of Egypt and Babylonia and give evidence of the activities of the last decade in the study of Hindu mathematics. Since the second editions of these volumes (1921), important studies of the Rhind Papyrus have been published by T. Eric Peet and by Chancellor Chace and his collaborators. Work has been done with the Moscow Papyrus. The Michigan Papyrus 620 has added to our knowledge of Greek algebra. In addition, many intensive studies have been made of special fields as, for example, the work of the late Dr. Wieleitner, *Über das x der Mathematiker*. In general, the additions are noted by the insertions of sentences or paragraphs rather than by the recasting of the text.

The first volume, *Rechnen*, treats numerals, computation, and certain applications to business problems. Beside the increased attention to the mathematics of Egypt and Babylonia already noted, we find that the table of the development of Hindu-Arabic numerals has been enlarged to three times its length in the second edition. It now includes numerals from the Aśóka inscriptions and from the Bahksali manuscript, and Gobar numerals from two undated Arabic manuscripts. It is unfortunate that the Herodianic Greek numerals were set in type instead of being in a cut, for the exigencies of the composition spoil the appearance of the numerals. Again and again in these two volumes, the reader wishes that Dr. Tropfke might have made the lavish use of cuts and facsimiles that the late Professor Cajori was allowed in his *History of Mathematical Notations*.

The second volume, *Allgemeine Arithmetik*, treats the name algebra, the development of the number concept, algebraic operations and their symbols, and logarithms. The question of the spelling of proper names has in general been met by using the standardized transliterations for the Greek and Hindu ones, and in using the form which those in our alphabet have in their own language. An exception appears in the case of Napier, who is consistently called "Neper." In spite of the "Neperschen Logarithmen," Napier would probably be the preferable spelling. A trivial slip is in spelling Barrow's first name as Isaak.

The author has given up the spelling of Fibonacci's *Liber Abaci* used in the second edition of the *Allgemeine Arithmetik*, and has taken the spelling *Liber Abbaci*, apparently without giving any reason for this change. He has kept to the date 1228, which is that of a revision of the work, instead of using the 1202 which appears in the *incipit*.

The slips in proofreading seem to be few. Professor Karpinski sometimes

appears with a y. An article by Dr. Gandz is given the title "The origine of the term algebre."

As was the case in the earlier editions, the index will appear in volume 7. It is possible, however, to make use of the index of the second edition, for in each of these parts of the third edition the author has included tables which show the correspondence between page numbers and footnote numbers in the second and third editions.

VERA SANFORD

The General Properties of Matter. By F. H. Newman and V. H. L. Searle. New York, Macmillan, 1933. 338 pp.

This is the second edition of a work first published in 1928. It plans to give a general introduction to physics and covers such topics as gravitation, gyroscopic motion, elasticity, surface tension, viscosity, kinetic theory of matter, Fourier's theorem, osmosis and diffusion, hydrodynamics and wave motion. On the whole the work is up-to-date and sound, but the mathematics is quite heuristic and sometimes unsound. For example the only purely mathematical chapter is one of eight pages devoted to Fourier's theorem, and in this is announced the theorem that any continuous function is equivalent to its Fourier series. The writers quite correctly state that a rigid proof of this (incorrect) theorem is beyond the scope of their book. The references in this chapter are quite out-of-date. In startling contrast is the immediately following fine chapter on osmosis and diffusion. We can recommend the work strongly to mathematicians who wish to find in compact and easily understandable form illustrations of the important applications of elementary mathematics to physical problems.

F. D. MURNAGHAN

Leçons d'Analyse Vectorielle. By Gustave Juvet. Paris, Gauthier-Villars, 1933. 120 pp.

This book contains the mathematical part of a course in vector analysis. A second volume containing the applications to physics will appear later. The subjects treated in the present volume are vector algebra, curves, surfaces, the differential and integral operators, and the analysis of scalar and vector fields.

The notation (dot, cross, inverted delta) is that of Gibbs. Vectors are distinguished from scalars by the use of arrows. In the proofs continuity and analytic character of functions are assumed and free use is made of geometrical intuition. The differential operators are defined by the integration method of Ignatowsky, the existence of the limit being assumed. Exercises are given at the end of each chapter.

The main purpose of vector analysis is to provide a notation and collect together a set of theorems which are found useful in the exposition of such subjects as theoretical electricity, hydrodynamics, theory of elasticity. For such use the little volume of M. Juvet may be heartily recommended.

H. B. PHILLIPS

A Study of the Traité des Indivisibles of Gilles Persone de Roberval. By Evelyn Walker. New York, Bureau of Publications, Teachers College, Columbia University, 1932. vi+272 pp.

According to the extended title, on the title-page, this study was undertaken "with a view to answering, insofar as is possible, the two questions: Which propositions contained therein are his own, and which are due to his predecessors or contemporaries? and, What effect, if any, had this work on his successors?"

The answers to these questions are found in a summary (pp. 165–167) at the end of Part 2 of the book. Roberval is credited, among other accomplishments, with having "invented a method of integration by means of infinitesimals"; "founded his treatment of infinitesimals upon an arithmetic basis, thereby anticipating Wallis"; "successfully applied his method of infinitesimals in order to establish a number of quadratures and cubatures, and to find the centers of gravity of certain plane and solid figures"; drawn "the first trigonometric graph that was ever constructed, namely, the curve of sines, found the area under it and the volume of the solid generated by revolving it about its base line as axis"; made various discoveries relating to the cycloid and, in so doing, "invented a new curve which he called the companion of the cycloid."

Two of the most important conclusions reached are given in the paragraphs numbered 10 and 12 (pp. 166–167) and they are here quoted in full:

"10. In this treatise, Roberval has given one example of his method of drawing tangents to curved lines by means of composition of motions, namely, in the case of the cycloid. The invention of the method gave him the right to be considered a pioneer in the domain of the differential calculus as well as in that of the integral calculus. A method almost identical with his was later used by Torricelli and still later by Wallis and Barrow. In Newton's hands it developed into the method of fluxions, and in those of Leibniz into the method of differentials.

"12. Furthermore, Roberval's influence as a teacher, as a member of Mersenne's Academy and later of the Académie Royale des Sciences, and finally as a correspondent of other scientists, was probably much more widespread than has been generally realized."

These statements seem slightly more conservative than the following paragraph, which appears on the wrapper:

"Gilles Persone de Roberval, who made important contributions to the early development of both the differential and the integral calculus, is rarely given the credit that is his due. He has, indeed, been treated with unpardonable neglect by historians of mathematics, although he was held in highest esteem by his contemporaries. The present work, in which some of his achievements are indicated, may help to restore him to his proper place among the scholarly pioneers of the seventeenth century."

Possibly the author placed in the text itself the conclusions which she felt had been sufficiently documented and on the more perishable jacket the somewhat more emphatic personal convictions to which the study had led her.

An "Acknowledgment" following the title-page states that the study was inspired and aided by Dr. David Eugene Smith. Evidently, during his years of exploration in the field of mathematical history, Dr. Smith, more or less in-

formally and gradually, came to some such conclusions as those stated above and, feeling that if they were accurate the evidence ought to be carefully collected and arranged, he suggested this work to Dr. Walker as a very worth while undertaking. The facts that Dr. Smith suggested the study and that he undoubtedly read and approved of the manuscript strengthens greatly the reader's confidence in Dr. Walker's conclusions.

An introduction of twenty-nine pages, which sketches the life and character of Roberval, tells of his relations with other mathematicians, and explains how he lost the credit for his discoveries, furnishes an excellent background for the discussion of his merits.

The marshalling of evidence and presentation of the main argument constitute Part II "Discussion of the Contents of the *Traité des Indivisibles*" (pp. 33–167). The author contrasts the work of Roberval with that of Cavalieri, discusses Roberval's treatment and invention of curves and his contributions to the processes of differentiation and integration, and considers the question of priority for his important discoveries.

Part III (pp. 171–258) incorporates the translation of the *Traité* itself and the author has made good her promised effort "to render clear the author's meaning, and to remove, as far as possible, the reproach of obscurity that has always attached to Roberval's writings."

Dr. Walker's style of writing is such as to inspire confidence. The reader soon comes to feel that much patient searching for material and careful consideration in weighing evidence have gone into the making of this book. She deserves recognition for having made a distinct and valuable contribution not only to our understanding and appreciation of the work of Roberval but also to our general perspective of the very interesting period in which he lived.

The addition of a ten page bibliography enhances materially the value of the book as a work of reference. The figures are beautifully drawn, and the work of the publishers has been well done.

U. G. MITCHELL

Les Théorèmes de Conservation dans la Théorie des Chocs Electroniques. By L. Goldstein. Paris, Hermann, 1933. 26 pp.

This is number 70 of the series *Actualités Scientifiques et Industrielles*, and number 9 of the sub-series *Exposés de Physique Théorique*, published under the direction of L. de Broglie. The monograph discusses the problem of electron-atom collisions by the method of Born and Dirac and arrives at the conclusion that the principle of conservation of energy and momentum is valid. A short résumé of the Born-Dirac method is given. A reader with a mathematical spirit will doubtless be confounded to meet at the beginning of an essential point of the argument the sentence "Nous ne manquerons pas de signaler que cette demonstration laisse à désirer du point de vue de la rigueur." The monograph concludes with some general remarks on proton-atom collisions and atom-atom collisions.

F. D. MURNAGHAN

Interpolation und genäherte Quadratur. By Dr. Gerhard Kowalewski. Leipzig, Teubner, 1932. 146 pp.

This book is a distinct addition to the theory of interpolation and approximate quadrature. The treatment is what we should expect of the author—a scholarly treatment of the theoretical side of the subjects. He lays particular stress upon the exact representation of the error term for which most books give only an estimate. The author stresses the importance of the Euler-Maclaurin formula, for which he gives a new derivation. This formula is often treated in a superficial manner in works on the calculus.

The subject is treated entirely by the calculus of differences and the results are given in a new form. The subjects treated are the following: 1. Interpolation by Polynomials. 2. Newton's Quadrature Formulas. 3. Euler-MacLaurin's and Boole's Theorems. 4. The Euler-Maclaurin's and Boole's Theorems as source of Quadrature Formulas.

The treatment is purely theoretical and will be of great value to persons investigating the subject. The author thinks the book will be of service to all who are concerned with applied mathematics, such as physicists, chemists, engineers, biologists, psychologists, and physiologists. The writer of this review is inclined to think that this will not be the case since few persons except mathematicians will be able to follow the rather abstract treatment of the author. Then, too, there is a dearth of examples (only three in the whole book), which is a great disadvantage to persons who are studying the subject more for its applications than for the theory.

In the second chapter the author considers what he terms "Newton's Favorite Formula," more commonly called "Simpson's Three-Eighths Rule" in the English books. Apparently he considers this formula to give more accurate results than Simpson's One-Third Rule—called by the author merely "Simpson's Rule." To show the comparative accuracy of the two rules he works out an example, $\int_0^{\pi} \sin x\,dx = 2$, by Simpson's Rule (with three ordinates), and by Newton's Favorite Formula (with four ordinates) and obtains the values $2\pi/3 = 2.09$ by the first formula and $3\pi\sqrt{3}/8 = 2.04$ by the second. However, it is evident that this is not a fair comparison. If we take seven ordinates for each formula, we should get by Simpson's Rule $\pi(4+\sqrt{3})/9 = 2.00086$ and by Newton's Favorite Formula $\pi(5+3\sqrt{3})/16 = 2.00201$, so that the error by the former rule is less than one-half that by the second.

The greater exactness of Simpson's Rule over the Three-Eighths Rule has been previously noted. See, for example my review of Scarborough's *Numerical Mathematical Analysis* (American Mathematical Monthly, vol. 38 (1931), pp. 396–402).

E. B. Escott

Origine des Rayons Gamma. Structure Fine du Spectre Magnétique des Rayons Alpha. By Salomon Rosenblum. (Exposés de Physique Théorétique, No. 4.) Paris, Hermann, 1932. 37 pp.

The memoir is one of a series of theoretical physical presentations by various authors and is edited by Louis de Broglie of the Sorbonne, a Nobel Laureate.

The author presents historically the subject of the study of magnetic spectra of alpha particles. In particular, he describes his own excellent researches with the large Academy of Science magnet on the fine structure of the magnetic spectra of alpha particles. He shows the correlations between the differences of energies of the various groups of alpha particles of several radioactive bodies and the γ-rays from these bodies which have otherwise been studied, and points out the agreement with Gamow's theory. Gamow predicted in his theory of the disintegration of a radioactive atom that γ-rays should be emitted in the case of those atoms for which the fine structure in the ranges exists, presupposing different energy levels in the nucleus. The emission of the α-particles from these levels accounts for the fine structure of the magnetic spectra of alpha rays. A fall of an α-particle to a lower level in the nucleus characterizes the quantum emitted as a gamma ray.

A complete bibliography is given.

A. F. KOVARIK

Cartesian Tensors. By Harold Jeffreys. Cambridge, The University Press, and New York, Macmillan, 1931. vi+92 pp.

This little book is devoted mostly to applied mathematics. The subject content is indicated by the chapter headings which are as follows: Cartesian Tensors, Geometrical Applications, Particle Dynamics, Dynamics of Rigid Bodies, Equivalence of Systems of Forces, Continuous Systems, Isotropic Tensors, Elasticity, and Hydrodynamics.

The discussion throughout is confined to a euclidean three-space and with reference to rectangular cartesian coordinate systems. Hence the tensor quantities are with reference to the group of orthogonal transformations. According to the preface, "the object of this work is to illustrate the use of tensor methods." In the opinion of the reviewer it is scarcely possible to accomplish this object when no distinction is made between covariant and contravariant tensors. The significance of tensors is necessarily missed unless the discussion is with respect to transformations of coordinates at least as general as the group of affine transformations. In slightly more space, the author could have given an introduction to tensor analysis useful to a beginner. Certain elementary but fundamental things such as a basis of a linear vector space, and the usual interpretations of the coefficients of contravariant and covariant vectors in terms of the base vectors and the reciprocal system, are most clearly presented by considering a euclidean three-space and the group of affine transformations. Then by specializing the transformations to the orthogonal group the formal results concerning tensors which the author gives would result, and at the same time the beginning student would see why the distinction between the two types of vectors disappears. As it is, the formal results concerning tensors which are developed are almost trivial to the student of tensor analysis, and at the same time, it seems to me, quite likely to be misleading to the uninitiated in the subject. However, to those interested in the topics of applied mathematics which are treated, and who know the elements of tensor analysis, the work should be of interest.

J. H. TAYLOR

NOTES

The Econometric Society, an international society for the advancement of economic theory in its relation to statistics and mathematics, has established a quarterly journal, entitled Econometrica, under the editorship of Ragnar Frisch, with H. T. Davis, A. H. Hansen, and F. C. Mills as associate editors. The editorial advisory board includes Irving Fisher, Harold Hotelling, Oystein Ore, and C. F. Roos. The business address of the journal is care of Alfred Cowles, Mining Exchange Building, Colorado Springs.

An account of the life and works of the late Professor J. C. Fields, of the University of Toronto, has been prepared by Professor J. L. Synge, of the same University, and published in the Journal of the London Mathematical Society (vol. 8, No. 2, pp. 153–160; April, 1933).

The fourth International Congress for Applied Mechanics will be held at Cambridge, England, on July 3–9, 1934. All persons interested in the subject are invited to attend and submit papers. It is suggested that the subjects be grouped as follows: rational mechanics, including vibrations of structures and machines; mechanics of fluids, including turbulence, the boundary layer, heat transfer, and compressible fluids; materials, including elasticity, plasticity, fatigue, and crystal structure; water waves, including resistance and stability of ships and seaplanes. Inquiries regarding the Congress and promises of manuscripts may be addressed to the Applied Mechanics Division of the American Society of Mechanical Engineers, 29 West 39th Street, New York City. Summaries, about 300 words in length, written in English, German, French or Italian, will be required by April 1, 1934, for all papers to be presented.

The officers of the Notgemeinschaft der Deutschen Wissenschaft have presented their resignations. They include Dr. Friedrich Schmidt-Ott, formerly "Kultus" minister, president since its foundation; Dr. Walter von Dyck, professor of mathematics at the Munich Institute of Technology, Professor Fritz Haber, who recently resigned as director of the Kaiser Wilhelm Institute for Physical Chemistry, and Dr. Heinrich Konen, professor of physics at Bonn, vice-presidents.

An Academic Assistance Council has been organized in London, with Lord Rutherford as chairman; its object is to provide positions or other aid for university professors and scientific men who because of "religion, political opinion, or race are unable to carry on their work in their own country." For substantially the same purposes, the Emergency Committee in Aid of Displaced German Scholars has been founded in the United States, with President Livingston Farrand, of Cornell, as chairman, and Dr. Stephen Duggan as secretary. This Committee includes also Professors R. A. Millikan, Harlow Shapley, and Oswald Veblen as members. It will be glad to receive contributions, which should be sent to the Treasurer, Mr. F. M. Stein, 2 West 45th Street, New York City; inquiries should be addressed to Dr. Duggan, at the same place.

At the request of the National Recovery Administration Dr. C. F. Roos has been granted an indefinite furlough by the Guggenheim Foundation to enable him to return to Washington to develop the postulates underlying the National Government's recovery programs,—industrial, agricultural, and banking. Dr. Roos will head a study which will have for its object the determination of demand and cost of production functions for the purpose of determining the effects of wage changes. In particular Dr. Roos will also develop dynamical theories of economics applicable to the new economy. He will be assisted by a number of mathematicians in Washington. Professors Harold Hotelling and J. A. Shohat will act in advisory capacities.

The Belgian Academy has awarded a subvention from its Wetrems foundation to Jules Géhéniaux, to assist him in continuing his researches on electromagnetic waves and Dirac's equation extended to interactions.

Cambridge University has awarded its Adams prize for 1931–32 to A. H. Wilson, research fellow of Trinity College.

Dr. Arnold Sommerfeld, professor of theoretical physics at Munich, has been awarded the James Scott prize of the University of Edinburgh for 1933.

The Reale Accademia dei Lincei has awarded its Royal Prize in mathematics jointly to Professors A. Comessatti, of the University of Padova, and L. Fantappiè, of the University of Bologna. The same Academy has awarded to Professor E. Bompiani, of the University of Rome, the prize founded by the Associazione Bancaria and for this year assigned to mathematics, and to Dr. Maria Cibraio, of Turin, and Professor P. Mazzoni, of Bari, the Ministerial Prizes in mathematics.

The Institute of Radio Engineers of New York has awarded its Gold Medal of Honor to Sir Ambrose Fleming, inventor of the thermionic valve, for the conspicuous part he has played in introducing physical and engineering principles into the art of the radio.

The Rumford medal of the American Academy of Arts and Sciences has been awarded to Professor Harlow Shapley, of Harvard University, for researches on the luminosity of stars and galaxies.

Dr. Ernst Zermelo and Dr. Gustav Doetsch, of the department of mathematics of the University of Freiburg, have been elected members of the Academy of Sciences of Heidelberg.

Professor Albert Einstein has been elected a foreign associate of the Paris Academy of Sciences, and Dr. Willem de Sitter a correspondent in the section of astronomy.

Professor P. Zeeman, of Amsterdam, has been elected associate of the Belgian Academy, in the section of mathematical and physical sciences.

The University of Dublin has conferred an honorary doctorate on Professor W. L. Bragg, of the University of Manchester.

Professors V. F. K. Bjerknes, of the University of Oslo, and Peter Debye, of the University of Leipzig, have been elected foreign members of the Royal Society of London.

The University of Edinburgh has conferred an honorary doctorate on Professor G. H. Hardy, of Cambridge.

At a special congregation of the University of British Columbia in honor of the Fifth Pacific Science Congress, an honorary doctorate was conferred on Dr. G. I. Taylor, Yarrow Research Professor of the Royal Society of London.

The University of Poitiers, on the occasion of the celebration of the 500th anniversary of the foundation of the University, on June 1, 1933, conferred the degree of Docteur Honoris Causa on Professor G. D. Birkhoff of Harvard University.

Harvard University has conferred honorary doctorates of science on Professors G. D. Birkhoff and Harlow Shapley, of its own faculty.

Brown University has conferred honorary doctorates on Professors Neils Bohr, of Copenhagen, and Lipót Fejér, of Budapest.

Yale University has conferred an honorary doctorate of science on Dr. E. W. Brown, Josiah Willard Gibbs professor of mathematics emeritus at that University.

Oglethorpe University has conferred an honorary doctorate on Professor Archibald Henderson, of the University of North Carolina.

The Maryland Academy of Sciences has conferred a diploma on Mr. L. B. Robinson, of Baltimore, "in recognition of his contributions to its Bulletin and his other distinguished services to mathematics."

Dr. David Eugene Smith, professor emeritus of mathematics at Columbia University, has been decorated by Persia for his study of the mathematical works and the philosophy of Omar Khayyam.

The University of Wisconsin has conferred an honorary doctorate on Professor Arnold Sommerfeld, of the University of Munich.

Dr. R. A. Fisher, head of the statistical department of the Rothamstead Experimental Station, has been appointed to the Galton chair of eugenics at the University of London.

Mr. P. Hall, of King's College, Cambridge, has been appointed to a Cambridge University lectureship in mathematics.

Dr. H. Jones has been appointed lecturer in theoretical physics at the University of Bristol.

Among those who will be in residence during the coming academic year in the school of mathematics of the Institute for Advanced Study, Princeton, are the following: A. A. Albert, of Chicago, W. E. Bleick, of Johns Hopkins; L. M. Blumenthal, of the Rice Institute; K. Gödel, of Vienna; K. Jessen, of Copenhagen; E. R. van Kampen, of Johns Hopkins; A. N. Lowan, of Columbia, T. S. Peterson, of California Institute of Technology; L. S. Pontrjagin, of Moscow; H. S. Ruse, of Edinburgh; M. Salkover, of Cincinnati; T. Y. Thomas, of Princeton; and R. L. Wilder of Michigan, (already announced in the July number of this Bulletin).

Assistant Professor J. G. Adshead, of Dalhousie University, has been promoted to an associate professorship.

Dr. J. V. Atanasoff, of Iowa State College, has been promoted to an assistant professorship.

Dr. G. D. Birkhoff, professor of mathematics at Harvard University, has been elected to the Perkins professorship of mathematics at that University, as successor to Professor W. F. Osgood, who retires as professor emeritus on September 1.

Assistant Professor H. W. Brinkmann, of Harvard University, has been appointed to an associate professorship at Swarthmore College.

Dr. Dirk Brouwer has been promoted to an assistant professorship of astronomy at Yale.

Professor E. W. Brown, of Yale University, has retired, as professor emeritus.

Associate Professor E. T. Browne, of the University of North Carolina, has been promoted to a professorship of mathematics.

Dr. L. P. Eisenhart, professor of mathematics and dean of the faculty of Princeton University, has been elected dean of the graduate school.

Dr. James Franck, formerly professor of physics at the University of Göttingen, has been appointed Speyer guest professor at Johns Hopkins University for the coming academic year.

Dr. J. J. Gergen, of Harvard University, has been appointed to an assistant professorship at the University of Rochester.

Professor M. W. Haskell, of the University of California, has retired, as professor emeritus, after forty-four years of service. Professor C. A. Noble will act as chairman of the department of Mathematics during the present academic year.

Alan Hazeltine has returned to the Stevens Institute of Technology as professor of physical mathematics.

Dr. I. O. Horsfall, of Cornell University, has been appointed president of Snow College, Ephraim, Utah.

Assistant Professor K. W. Lamson, of Lehigh University, has been promoted to an associate professorship of mathematics.

Assistant Professor A. E. Meder, of the New Jersey College for Women, has been promoted to an associate professorship of mathematics.

Professor Helen A. Merrill, of Wellesley College, has retired, as professor emeritus.

Mr. F. H. Miller, of Columbia University, has been appointed to an assistant professorship at Cooper Union.

Professor B. P. Reinsch, of the Southern Methodist University, Dallas, has been appointed professor and head of the department of mathematics at Southern College, Lakeland, Florida.

Miss Vera Sanford has been appointed professor of mathematics at the Oneonta (N.Y.) State Normal School.

Assistant Professor C. N. Shuster, of the New Jersey State Teachers College, Trenton, has been promoted to a professorship.

Professor Mary C. Spencer, of Newcomb College, has retired, as professor emeritus.

Mr. G. A. Yanosik has been promoted to an assistant professorship at New York University.

The following appointments to instructorships in mathematics are announced:
University of California (Berkeley): Dr. A. L. Foster, Dr. C. B. Morrey, Jr.;
Sweet Briar College: Miss Nancy Cole;
Vassar College: Mrs. Grace Murray Hopper.

Professor Piero Benedetti, of the Pisa Technical Institute, died March 17, 1933.

Professor Giovanni Bordiga, of the University of Padova, died June 16, 1933.

S. A. Gerschgorin, professor of theoretical mechanics at the Institute for Mechanical Engineering, Leningrad, died May 30, 1933, at the age of thirty-two.

Professor Duilio Gigli, of the Reale Liceo Foscolo and the University of Pavia, died May 10, 1933.

Professor Lebrecht Henneberg, of the department of mechanics at the Darmstadt Technical School, died April 29, 1933. at the age of eighty-two.

Miss Vevia Blair, head of the department of mathematics at Horace Mann School for Girls, an observation school of Teachers College, Columbia University, died July 9, 1933, at the age of sixty-two. Miss Blair had been a member of the American Mathematical Society since 1920.

Professor E. F. Miller, head of the department of mechanical engineering at the Massachusetts Institute of Technology, died June 12, 1933, at the age of sixty-seven.

ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

239. Professor A. A. Albert: *Normal division algebras over algebraic number fields not of finite degree.*

The author proves here that every normal division algebra of degree n over an algebraic number field not of finite degree is cyclic and has exponent n. This theorem was known only for algebraic number fields of finite degree. By a trivial example he also shows that there exist non-cyclic normal simple algebras over algebraic number fields not of finite degree (while this is known not to be true over fields of finite degree). He finally considers also the problem of the equivalence of normal division algebras, reducing the problem for the fields here considered to the already solved problem for the case of finite degree. (Received July 22, 1933.)

240. Professor E. T. Bell: *Exponential numbers.*

An examination of some faulty physical calculations suggested the following problem. Let the power series expansion of $f(x)$ and the MacLaurin expansion of $\exp f(x)$ be absolutely convergent and termwise differentiable. It is required to devise some readily applicable arithmetical check on the accuracy of the heavy algebra usually involved in carrying the MacLaurin expansion to more than ten terms. This is provided in the present paper by congruences which the coefficient of $x^n/n!$ $(n=1, 2, \cdots)$ in the MacLaurin expansion of $\exp[f(x)-f(0)]$ must satisfy. Applications of the criteria disclose errors in the tabulated expansions, some of which have been copied from one handbook into several later compilations. (Received July 31, 1933.)

241. Professor E. T. Bell: *Polynomial diophantine systems.*

The method of reciprocal arrays, developed by the author for purely multiplicative diophantine systems, is here extended to diophantine systems consisting of any number of polynomials, in any numbers of indeterminates, of any degrees. Such systems as are solvable by the method are *completely* solvable (*all* sets of integers satisfying the system are given explicitly in parametric form). The theory is first developed for an abstract commutative ring. Application is then made to diophantine polynomial systems in any algebraic number field with class number unity. (Received July 21, 1933.)

242. Professor H. R. Brahana: *On cubic congruences.*

Any two irreducible cubic congruences with coefficients in the same modu-

lar field (the modulus being a prime) are conjugate under a linear fractional transformation on the variable with coefficients in the same field. If $x^3-x+\beta \equiv 0$ is irreducible, then $x^3-x+\beta(4-27\beta^2)^{1/2}\equiv 0$ is also irreducible. If p is of the form $6k+1$ and $x^3+\beta\equiv 0$ is irreducible, then $x^3-\alpha x+(27\beta^2+c^6\alpha^3)/(27\beta c^3) \equiv 0$, α and c being any numbers from 1 to $p-1$, is irreducible, and every irreducible cubic with coefficient of x^2 equal to zero can be written in that form with β the same for all of them. (Received July 10, 1933.)

243. Professor H. R. Brahana: *On the isomorphisms of an abelian group of type* 1, 1.

The operators of order a power of p in the group of isomorphisms of the abelian group H of order p^n and type 1, 1, $\cdots$ belong to conjugate sets each of which is completely characterized by a partition $n=n_1+n_2+\cdots+n_s$. Each such operator determines a subgroup of the holomorph of H whose characteristics (order, class, order of the central, order of the commutator subgroup, and order of the cross-cut of the central and commutator subgroup) are immediately expressible in terms of the numbers n_i. (Received July 10, 1933.)

244. Professor H. R. Brahana: *On the metabelian groups which contain a given group as a maximal invariant abelian subgroup.*

This paper is the beginning of a classification of the abelian subgroups of order p^m and type 1, 1, $\cdots$ of the group of isomorphisms of the abelian group of order p^n and type 1, 1, $\cdots$. We consider only those subgroups whose operators are of two types, namely those corresponding to the partitions $n=2+1+1+\cdots+1$ and $n=2+2+1+\cdots+1$. Of the groups which contain at least one operator of the second type there are 6 types of order p^3 and 17 types of order p^3. (Received July 10, 1933.)

245. Dr. Rothwell Stephens: *Note on a problem of Fréchet.*

Fréchet has proposed the following problem: Characterize the most general space in which there exists a non-constant continuous function. Its solution has been given by Urysohn for the spaces of Hausdorff, and by Chittenden for topological space. However, in his generalization Chittenden used for his definition of a continuous function a neighborhood definition which is not entirely adequate for general topological spaces. The solution of the problem using the Sierpinski definition of a continuous transformation is given in the following theorem: A necessary and sufficient condition that a topological space admit a non-constant continuous function is that it contain a normal family of thoroughly open sets and that it contain no singular points. (Received July 18, 1933.)

246. Professor R. L. Wilder: *Concerning a problem of K. Borsuk.*

K. Borsuk has raised a question (Fundamenta Mathematicae, vol. 20, p. 285, problem 54) as to whether a subcontinuum C of E_n which cuts E_n, and which is ϵ-transformable into a set C' such that $C\cdot C'=0$, is an $(n-1)$-manifold. Although, as shown by examples, the answer to this question is negative (even if C is a Jordan continuum), several positive results are obtained. Thus (we assume C compact throughout) the complement of C is just two domains

with C as common boundary, and if C is a Jordan continuum, both these domains are uniformly locally connected; in the latter case, if $n=2$, C is a simple closed curve, and if $n=3$ and $p^1(C)$ is finite, C is a closed 2-dimensional manifold. If a continuum C that cuts E_n is deformable without meeting itself, then its complement is two uniformly locally connected domains and for $n=2, 3$, C is a closed $(n-1)$-manifold. One by-product of these investigations is a sharpening of a recent theorem of ours (this Bulletin, vol. 37, p. 525, Abstract No. 258): In order that a compact continuum C in E_3 should be a closed 2-dimensional manifold, it is necessary and sufficient that it be a common boundary of at least two uniformly locally connected domains for one of which there exists an $\epsilon>0$ such that none of its 1-cycles of diameter $<\epsilon$ links C. (Received June 24, 1933.)

247. Mr. J. A. Clarkson: *On double Riemann-Stieltjes integrals.*

We consider two definitions of double Riemann-Stieltjes integrals: that of the "restricted" integral is due to Fréchet (Nouvelles Annales, (4), vol. 10 (1910); Transactions of this Society, vol. 16 (1915); that of the "unrestricted" integral has recently been employed by Schoenberg and others (Transactions and Bulletin of this Society, 1933). The existence of the unrestricted integral for a particular integrator and integrand clearly implies the existence of the restricted integral (as well as the equality of their values); we show that the converse is not true. Fréchet has shown (a) that a sufficient condition for the existence of the restricted integral for every continuous integrand is that the integrator be of bounded variation in the Vitali sense; and (b) that a sufficient condition for the existence of the restricted integral for every integrand which is the product of a continuous function of x and a like function of y is that the integrator be of bounded variation in the Fréchet sense. We prove the necessity of the condition in both (a) and (b). We observe that in case (a) the condition specified is also sufficient for the existence of the unrestricted integral, and prove that in case (b) the condition stated is not sufficient to insure the existence of the unrestricted integral. (Received August 10, 1933.)

248. Professor R. P. Agnew: *On summability of multiple sequences.*

The author extends to sequences of arbitrary multiplicity Theorem 1 of his paper *On summability of double sequences.* American Journal of Mathematics, vol. 54 (1932), pp. 648–656, and hence obtains extensions of results of C. R. Adams, F. Lösch, and himself on summability of double sequences. (Received September 6, 1933.)

249. Professor P. A. Caris: *Integral solutions of* $(u^4-v^4)\cdot(x^4-y^4)=z^4-t^4$.

The diophantine equation $P^4+kQ^4=R^4+kS^4$, when solvable for a given k, implies that k can be represented as the quotient of two differences of bi-quadrates. It is here proved that it can be solved when k is itself the difference of two bi-quadrates. Thence are deduced integral solutions of the equation $(u^4-v^4)(x^4-y^4)=z^4-t^4$. A two-parameter solution is given by $u=a+b$, $v=a-b$, $x=a^4+18a^2b^2+13b^4$, $y=3(a^2-b^2)(a^2+3b^2)$, $z=2(a+b)(5b^4+6ab^3$

$+6a^3b-a^4)$, $t=2(a-b)(a^4+6a^3b+6ab^3-5b^4)$, where a, b are arbitrary integers. (Received September 2, 1933.)

250. Mr. W. R. Church: *Tables of irreducible polynomials for the first four prime moduli.*

The author gives tables listing irreducible polynomials $\phi(x)$, modulo p, together with their exponents, that is, the order of the element x in the finite field defined by $\phi(x)$, within the following limits: for $p=2$, up to and including degree 11; for $p=3$, up to and including degree 7; for $p=5$, up to and including degree 5; for $p=7$, up to and including degree 4. (Received September 7, 1933.)

251. Dr. J. M. Clarkson: *An involutorial line transformation.*

Given a quadric H, a plane π not tangent to H, and a point O on H but not on π. In the plane π, consider a Cremona involutorial transformation I_n of order n. An arbitrary line t meets H in two points A_1, A_2. Project H from O upon π by the projection P. The points A_1, A_2 are carried into A_1', A_2' by the transformation PI_nP^{-1}. The line $t'\equiv A_1'A_2'$ shall be called the conjugate of t by the line transformation T; t is carried into t' by the transformation T. The order, singular lines, and invariant lines of T are discussed. (Received September 5, 1933.)

252. Professor G. C. Evans: *Inferior limit of Newtonian potential at points of mass from points of empty space.*

Given in space of three dimensions a closed, bounded, reduced set t, which is the boundary of an unbounded region T and possibly also of other regions which constitute an open set B, and bears an arbitrary distribution of positive mass, finite in total amount, of which the Newtonian potential at M is $V(M)$, the problem deals with the validity of the equation (1): inf lim $(M=Q)V(M)=V(Q)$, Q in t, M in $T+B$. It is valid, if in the neighborhood of Q, t does not contain a collection of rectangular boundaries, with sides parallel to two fixed but arbitrary normal directions, whose vertices constitute a set of positive spatial measure. In particular, for the logarithmic potential, we have the topological situation that (1) holds if t is closed and bounded, and B is vacuous. But it is shown by an example that the corresponding topological theorem is not valid in three dimensions. The problem is related to various questions of approximation and unicity, in particular to the uniqueness of capacity distributions the upper bound of whose potential is unity. (Received August 26, 1933.)

253. Mr. F. B. Jones: *Concerning normal and completely normal spaces.*

The author shows that if a separable Fréchet space-L is normal, every point set of power c has a limit point, and if it is completely normal, every point set of power c contains one of its limit points. If $\aleph_1=c$, there exists an example of a normal separable Fréchet space-L in which every uncountable point set has a limit point, but in which there exists an uncountable point set that contains none of its limit points. Furthermore, a separable normal space satisfying the first three parts of Axiom 1 of R. L. Moore's *Foundations of Point Set Theory* is completely separable and therefore metric, provided that in this

space every uncountable point set contains a subset of power c. Also, a normal space satisfying the first three parts of Axiom 1 is completely normal. (Received August 26, 1933.)

254. Mr. F. B. Jones: *Concerning R. L. Moore's Axiom* 5_1. Preliminary report.

Consider the following axioms. Axiom 5_1*: *If P is a point of a region R, there exists in R a domain D containing P whose boundary is a subset of the sum of a finite number of continua lying in $R-D$. Axiom* $\bar{5}_1$: *If A is a point of a region R and B is a point distinct from A, there exists in R a closed and compact point set M separating A from B.* The author shows that if S is a non-acyclic space satisfying Axiom 5_1* together with Axioms 0, 1, 2, and 4 of R. L. Moore's *Foundations of Point Set Theory* (American Mathematical Society Colloquium Publications, vol. 13, 1932) then (1) space is cyclically connected, (2) if C, the boundary of a connected domain D, is a compact continuous curve, then every point of C is accessible from D, (3) Theorems 10, 12, 14, 16, 18, 19, 21, 22, 23, 24, 25, and a number of the remaining theorems of Chapter IV of the above work hold true. Furthermore, from Axiom $\bar{5}_1$ together with Moore's Axioms 0, 1, 2, and 4 it follows that either space is acyclic or Moore's Axiom 5 holds true. (Received August 26, 1933.)

255. Professor R. L. Moore: *Concerning compact continua which contain no continuum that separates the plane.*

Let S denote a definite compact plane continuum containing no continuum that separates the plane. For every two distinct points X and Y of S, let XY denote the irreducible subcontinuum of S from X to Y. In this paper the following theorems are established: I. If A and B are points of S, then (a) there exists a point O of S such that AO contains AB but is not a proper subset of AZ for any point Z of S, (b) if XY contains a continuum AO satisfying these conditions, then either $XY=OX$ or $XY=OY$, (c) there exists a point C such that OC contains AB but is not a proper subset of any irreducible subcontinuum of S. II. In order that S should be triodic it is necessary and sufficient that there should exist three points of S such that no one of them separates the remaining two from each other in S in the weak sense. III. If S is non-degenerate and atriodic, it is irreducible between some two of its points. (Received September 5, 1933.)

256. Dr. A. F. Moursund: *On summation of derived series of the conjugate Fourier series.*

The principal result of this paper is a theorem concerning the summability of the rth, $r \geqq 0$, derived series of the conjugate Fourier series. The author has been unable to find in the literature a single such theorem for the case $r>1$. A limit $\bar{f}^{(r)}(x)$ involving generalized derivatives (in the sense of de la Vallée Poussin) of orders $r-1$, $r-3$, $\cdots$, is defined; sufficient conditions for the existence of $\bar{f}^{(r)}(x)$ at a point and almost everywhere in an interval are given; and it is shown that the rth, $0 \leqq r \leqq p-1$, derived series of the conjugate Fourier series of a Lebesgue integrable function $f(x)$ is summable by the N_{2p} method (defined by the author in a paper to appear in the Annals of Mathematics) to $\bar{f}^{(r)}(x)$ wherever that limit exists. The N_{2p} method includes as a

special case the second form of the Bosanquet-Linfoot (α, β) method, $\alpha=p$, $\beta>1$ or $\alpha>p$ (Quarterly Journal of Mathematics, Oxford Series, vol. 2 (1931), pp. 207–229), which when $\alpha=p+\delta$, $\delta>0$, and $\beta=0$ is the Riesz equivalent of the Cesàro method $(C, p+\delta)$, $\delta>0$. (Received September 2, 1933.)

257. Dr. R. E. A. C. Paley and Professor Norbert Wiener: *Notes on the theory and application of Fourier transforms.* V: *On entire functions.*

The authors give an extension by general Tauberian methods of Titchmarsh's theory of the properties of entire functions of semi-exponential type with real negative zeros. (Received August 23, 1933.)

258. Dr. R. E. A. C. Paley and Professor Norbert Wiener: *Notes on the theory and application of Fourier transforms.* VI: *On two problems of Pólya.*

The authors answer and discuss two problems on entire functions set by Pólya in the Jahresbericht for 1931. (Received August 23, 1933.)

259. Dr. R. E. A. C. Paley and Professor Norbert Wiener: *Notes on the theory and application of Fourier transforms.* VII: *On the Volterra equation.*

By an extension to Fourier integrals of a theorem of Wiener concerning absolutely convergent Fourier series, the authors prove a generalization of Mercer's theorem concerning the equivalence of the convergence of a series and of an average between its ordinary and its Cesàro sum. (Received August 23, 1933.)

260. Dr. R. E. A. C. Paley and Professor Norbert Wiener: *Notes on the theory and application of Fourier transforms.* VIII: *On the closure of sets of complex exponential polynomials.*

The authors discuss the theory of the closure of non-harmonic sets of complex exponential functions. (Received August 23, 1933.)

261. Dr. R. E. A. C. Paley and Professor Norbert Wiener: *Notes on the theory and application of Fourier transforms.* IX: *On non-harmonic Fourier series.*

The authors show that if a non-harmonic Fourier series is nearly enough harmonic, in the sense that the maximum difference between n and the nth frequency is small enough, its convergence in the mean, convergence, and summability theories are substantially identical with those of ordinary Fourier series. (Received August 23, 1933.)

262. Dr. A. H. Smith: *On the summability of derived conjugate series of the Fourier-Lebesgue type.*

This note, which is an extension of the author's paper *On the summability of derived series of the Fourier-Lebesgue type* (Quarterly Journal of Mathematics, Oxford Series, vol. 4 (1933), pp. 93–106), applies the method of summation

introduced by Bosanquet and Linfoot (Journal of the London Mathematical Society, vol. 6 (1931), pp. 117–126) to the rth derived conjugate series of the Fourier series generated by a periodic function integrable in the sense of Lebesgue. (Received August 30, 1933.)

263. Dr. Hassler Whitney (National Research Fellow): *Differentiable functions defined in closed sets.* I.

Let A be a closed set on the x-axis E, and let $f(x)$ be defined in A. If $f(x)$ is "of class C^m in A," its definition can be extended over E so that it will have a continuous mth derivative there (see this Bulletin, January, 1933, p. 31). If $x_0, \cdots, x_m$ are distinct points of A, and $P(x)=c_0+\cdots+c_mx^m$ is the polynomial of degree at most m such that $P(x_i)=f(x_i)$ $(i=0, \cdots, m)$, define the difference quotient $\Delta(x_0, \cdots, x_m)=m!c_m$. It is shown that a necessary and sufficient condition that $f(x)$ be of class C^m in A is that for each point x of A and every $\epsilon>0$ there exist a $\delta>0$ such that if $x_0, \cdots, x_m$ and $x_0', \cdots, x_m'$ are any two sets of distinct points of A within δ of x_0, then $|\Delta(x_0', \cdots, x_m') - \Delta(x_0, \cdots, x_m)| < \epsilon$. Taylor's formula in finite form is discussed. (Received September 8, 1933.)

264. Dr. G. T. Whyburn: *Concerning separating points.*

A point p of a metric space M is a separating point of M provided $M-p$ is separated between some pair of points of the components of M containing p. A point which is a separating point of some open set in M is called a local separating point of M. In this paper the principal properties of these types of points previously known for continua are developed for locally compact metric spaces in general. The treatment shows conclusively that the notion of a local separating point is a true localization of that of a separating point of a closed set rather than that of a continuum, to which it was formerly restricted. Incidentally it is shown that if a closed set X separates a component C of a compact metric space K into two separated sets C_a and C_b, then X will also separate C_a and C_b in K if, and only if, the limit of no convergent sequence of components of K intersects both C_a and C_b. (Received September 5, 1933.)

265. Dr. G. T. Whyburn: *Concerning maximal sets.*

Let T be any monotone system of closed subsets of an arbitrary metric space M, i.e., any closed subset of a T-set is itself a T-set. It is proved in this paper that if N is any non-degenerate subset of M such that N is not disconnected by the removal of any T-set, then there exists a maximal subset $H(N)$ of M containing N and having this property. Furthermore the sets $H(N)$ are continua, and the common part of any two of them is a T-set. By taking particular systems T, a large number of different types of maximal sets are obtained, some of which are already known. For example, if T is the system of all closed sets of dimension $\leqq n-2$, we obtain for the sets $H(N)$ the so-called "n-dimensional components" of M. Also it is shown that if T is the system of all closed sets which contain no essential n-dimensional complete cycle, the sets $H(N)$ have the property that if K is any subcontinuum of M such that every n-cycle in K is ~ 0 in K, then every n-cycle in $K \cdot H(N)$ is ~ 0 in $K \cdot H(N)$. (Received September 5, 1933.)

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PART II. APPLIED MATHEMATICS

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OFFICIAL COMMUNICATIONS

Meetings of the Society have been fixed at the following times and places:

NEW YORK CITY, October 28, 1933.

Abstracts must be in the hands of Associate Secretary J. R. Kline, 501 West 116th St., New York City, not later than October 7. Professor R. L. Jeffery will deliver an address on *Theories of integration*. Abstracts received by the Secretaries by September 9 will appear in the September issue of the BULLETIN.

CINCINNATI, OHIO, December 1-2, 1933.

Abstracts must be in the hands of Associate Secretary M. H. Ingraham, University of Wisconsin, Madison, Wis., not later than November 11. Abstracts received by the Secretaries by November 4 will appear in the November issue of the BULLETIN.

PASADENA, CALIFORNIA, December 2, 1933.

Abstracts must be in the hands of Associate Secretary T. M. Putnam, University of California, Berkeley, Calif., not later than November 11. Abstracts received by the Secretaries by November 4 will appear in the November issue of the BULLETIN.

CAMBRIDGE, MASSACHUSETTS, ANNUAL MEETING, December 27-29, 1933.

This meeting is to be held in conjunction with the A.A.A.S. and the M.A.A. There will be a symposium on *General Analysis*. The Bôcher Prize will be awarded. Abstracts must be in the hands of the Secretary of the Society, 501 West 116th St., New York City, not later than November 30, 1933. Abstracts received by the Secretaries by November 4 will be published in the November issue of the BULLETIN.

R. G. D. RICHARDSON, *Secretary of the Society.*

Articles for insertion in the BULLETIN should be addressed to E. R. HEDRICK, Editor of the BULLETIN, University of California at Los Angeles. Reviews should be sent to W. R. LONGLEY, Yale University, New Haven, Conn. Notes should be sent to H. W. KUHN, Ohio State University, Columbus, Ohio.

Subscriptions to the BULLETIN, orders for back numbers, and inquiries in regard to non-delivery of current numbers should be addressed to the American Mathematical Society, 450-459 Ahnaip St., Menasha, Wis., or 501 West 116th St., New York.

The initiation fees and the annual dues of members of the Society (see this BULLETIN, p. 322, May, 1930; and the List of Officers and Members, September, 1932, p. 66), are payable to the Treasurer of the Society, Professor G. W. Mullins, 501 West 116th St., New York City.

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